

LEARNING MATHEMATICS FOR TEACHING

***MATHEMATICAL KNOWLEDGE FOR  
TEACHING (MKT) MEASURES***

**MATHEMATICS RELEASED ITEMS  
2008**

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December 26, 2008

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement (SII)*. SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President<sup>1</sup>, we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching (LMT)*. With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge. In other words, one cannot calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If

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<sup>1</sup> Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at [gphelps@umich.edu](mailto:gphelps@umich.edu). Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (<http://www.sii.soe.umich.edu/>) or LMT website (<http://www.sitemaker.umich.edu/lmt>) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

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**Study of Instructional Improvement/Learning Mathematics for Teaching**  
Content Knowledge for Teaching Mathematics Measures (MKT measures)  
Released Items, 2008

ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Ms. Chambreaux's students are working on the following problem:

**Is 371 a prime number?**

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
- c) Check to see whether 371 is divisible by any prime number less than 20.
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75\phantom{0} \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

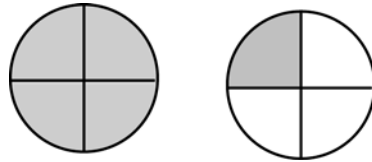
Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

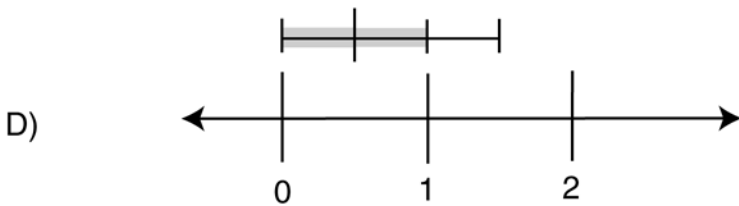
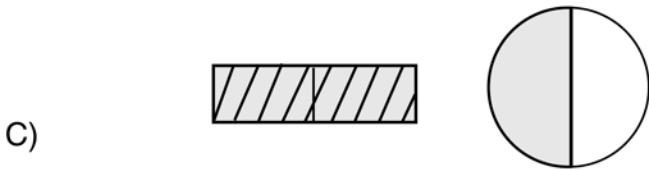
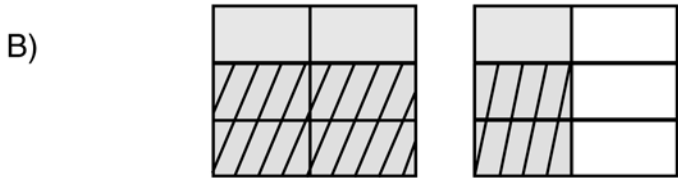
5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



- a)  $\frac{5}{4}$
- b)  $\frac{5}{3}$
- c)  $\frac{5}{8}$
- d)  $\frac{1}{4}$

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that  $1\frac{1}{2} \times \frac{2}{3} = 1$ ? (Mark ONE answer.)



7. Which of the following story problems could be used to illustrate  $1\frac{1}{4}$  divided by  $\frac{1}{2}$ ? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3



8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \times 6 \\ \hline 488 \\ +5410 \\ \hline 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

- a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.
- b) Todd is using the traditional multiplication algorithm but working from left to right.
- c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.
- d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

9. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

*I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.*

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

- a) It provides a general and efficient basis for the claim.
- b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.
- c) It only shows that the claim is true for one example, rather than establishing that it is true in general.
- d) It assumes what it is trying to show, rather than establishing why the sum is odd.

## KNOWLEDGE OF CONTENT AND STUDENTS ITEMS

10. Mr. Garrett’s students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to  $8 \times 8$ ? (Mark YES, NO, or I’M NOT SURE for each strategy.)

	Yes	No	I’m not sure
a) They might multiply $8 \times 4 = 32$ and then double that by doing $32 \times 2 = 64$ .	1	2	3
b) They might multiply $10 \times 10 = 100$ and then subtract 36 to get 64.	1	2	3
c) They might multiply $8 \times 10 = 80$ and then subtract $8 \times 2$ from 80: $80 - 16 = 64$ .	1	2	3
d) They might multiply $8 \times 5 = 40$ and then count up by 8’s: 48, 56, 64.	1	2	3

11. Students in Mr. Hayes’ class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

- a) They are ignoring place value.
- b) They are ignoring the decimal point.
- c) They are guessing.
- d) They have forgotten their numbers between 0 and 1.
- e) They are making all of the above errors.

12. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

- a) Bonny doesn't know how large 23 is.
- b) Bonny thinks that 2 and 20 are the same.
- c) Bonny doesn't understand the meaning of the places in the numeral 23.
- d) All of the above.

13. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I) $\begin{array}{r} 1 \\ 38 \\ 49 \\ + 65 \\ \hline 142 \end{array}$	II) $\begin{array}{r} 1 \\ 45 \\ 37 \\ + 29 \\ \hline 101 \end{array}$	III) $\begin{array}{r} 1 \\ 32 \\ 14 \\ + 19 \\ \hline 64 \end{array}$
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Which have the same kind of error? (Mark ONE answer.)

- a) I and II
- b) I and III
- c) II and III
- d) I, II, and III

14. Ms. Walker’s class was working on finding patterns on the 100’s chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e.,  $22 + 32 + 42 = 31 + 32 + 33$ ). Which of the following student explanations shows sufficient understanding of why this is true for all similar plus signs? (Mark YES, NO or I’M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I’m not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

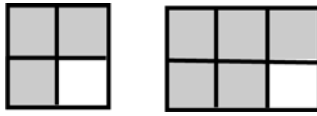
15. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I	II	III
$\begin{array}{r} 4 \ 12 \\ \cancel{502} \\ - 6 \\ \hline 406 \end{array}$	$\begin{array}{r} 4 \ 15 \\ \cancel{3000} \\ - 6 \\ \hline 34009 \end{array}$	$\begin{array}{r} 6 \ 9 \ 8 \ 15 \\ \cancel{7000} \\ - 7 \\ \hline 6988 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

- a) I and II
- b) I and III
- c) II and III
- d) I, II, and III

16. Takeem's teacher asks him to make a drawing to compare  $\frac{3}{4}$  and  $\frac{5}{6}$ . He draws the following:



and claims that  $\frac{3}{4}$  and  $\frac{5}{6}$  are the same amount. What is the most likely explanation for Takeem's answer? (Mark ONE answer.)

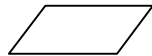
- a) Takeem is noticing that each figure leaves one square unshaded.
- b) Takeem has not yet learned the procedure for finding common denominators.
- c) Takeem is adding 2 to both the numerator and denominator of  $\frac{3}{4}$ , and he sees that that equals  $\frac{5}{6}$ .
- d) All of the above are equally likely.

17. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because  $1 + 2 + 3 + 4 + 6 > 12$ . On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

	Yes	No	I'm not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that "abundant" is another name for square numbers.	1	2	3

18. At the close of a lesson on reflection symmetry in polygons, Ms. White gave her students several problems to do. She collected their answers and read through them after class. For the problem below, several of her students answered that the figure has two lines of symmetry and several answered that it has four.

How many lines of symmetry does this figure have?



Which of the following is the most likely reason for these incorrect answers? (Circle ONE answer.)

- a) Students were not taught the definition of reflection symmetry.
- b) Students were not taught the definition of a parallelogram.
- c) Students confused lines of symmetry with edges of the polygon.
- d) Students confused lines of symmetry with rotating half the figure onto the other half.

19. Ms. Abdul is preparing a unit to introduce her students to proportional reasoning. She is considering three versions of a problem that are the same except for the numbers used. Which version of the Mr. Short and Mr. Tall problem below is likely to be the most challenging for students? (Circle ONE answer.)

- a) A picture depicts Mr. Short's height as 4 paper clips and as 6 buttons. The height of Mr. Tall (not shown) is given as 6 paper clips. How many buttons in height is Mr. Tall?
- b) A picture depicts Mr. Short's height as 4 paper clips and as 7 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?
- c) A picture depicts Mr. Short's height as 2 paper clips and as 9 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?
- d) All three of the problems are equally challenging.

#### ELEMENTARY AND MIDDLE SCHOOL KNOWLEDGE OF CONTENT AND TEACHING ITEMS

20. To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Circle ONE answer.)

- a) A number line
- b) Plastic counting chips
- c) Pennies and dimes
- d) Straws and rubber bands
- e) Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.



21. Mr. Foster's class is learning to compare and order fractions. While his students know how to compare fractions using common denominators, Mr. Foster also wants them to develop a variety of other intuitive methods.

Which of the following lists of fractions would be best for helping students learn to develop several different strategies for comparing fractions? (Circle ONE answer.)

a)  $\frac{1}{4}$   $\frac{1}{20}$   $\frac{1}{19}$   $\frac{1}{2}$   $\frac{1}{10}$

b)  $\frac{4}{13}$   $\frac{3}{11}$   $\frac{6}{20}$   $\frac{1}{3}$   $\frac{2}{5}$

c)  $\frac{5}{6}$   $\frac{3}{8}$   $\frac{2}{3}$   $\frac{3}{7}$   $\frac{1}{12}$

d) Any of these would work equally well for this purpose.

22. Ms. Brockton assigned the following problem to her students:

How many 4s are there in 3?

When her students struggled to find a solution, she decided to use a sequence of examples to help them understand how to solve this problem. Which of the following sequences of examples would be best to use to help her students understand how to solve the original problem? (Circle ONE answer.)

a) How many:

4s in 6?

4s in 5?

4s in 4?

4s in 3?

b) How many:

4s in 8?

4s in 6?

4s in 1?

4s in 3?

c) How many:

4s in 1?

4s in 2?

4s in 4?

4s in 3?

d) How many:

4s in 12?

4s in 8?

4s in 4?

4s in 3?

23. Ms. Williams plans to give the following problem to her class:

Baker Joe is making apple tarts. If he uses  $\frac{3}{4}$  of an apple for each tart, how many tarts can he make with 15 apples?

Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem? (Circle ONE answer.)

- I. Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses  $\frac{1}{4}$  of his pumpkins per pie, how many pumpkins does he use in each pie?
- II. Baker Ted is making pumpkin pies. If he uses  $\frac{1}{4}$  of a pumpkin for each pie, how many pies can he make with 9 pumpkins?
- III. Baker Ted is making pumpkin pies. If he uses  $\frac{3}{4}$  of a pumpkin for each pie, how many pies can he make with 10 pumpkins?


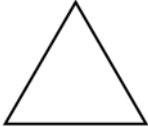
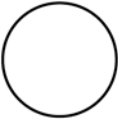

- a) I only
- b) II only
- c) III only
- d) II and III only
- e) I, II, and III

24. Ms. Miller wants her students to write or find a definition for triangle, and then improve their definition by testing it on different shapes. To help them, she wants to give them some shapes they can use to test their definition.

She goes to the store to look for a visual aid to help with this lesson. Which of the following is most likely to help students improve their definitions? (Circle ONE answer.)

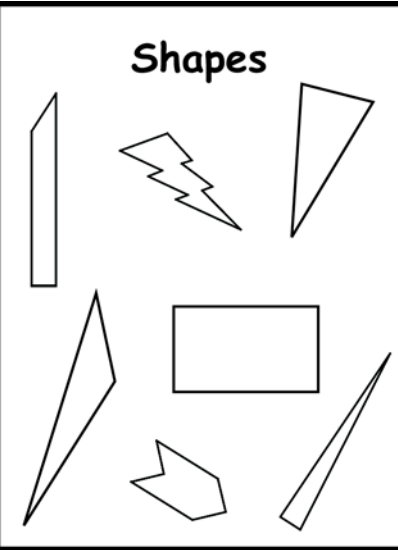
a)

**Shapes**

 square	 triangle
 circle	 rectangle

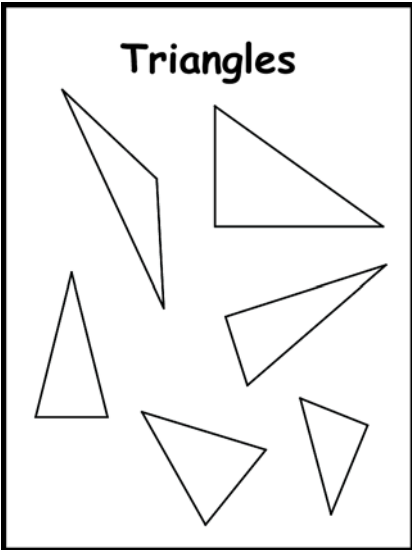
b)

**Shapes**




c)

**Triangles**



d)

**Triangles**

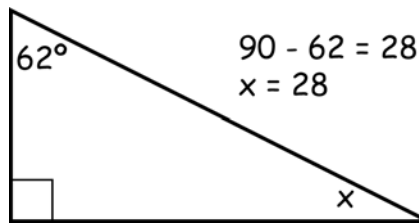


A triangle has 3 corners, 1 on the top and 2 on the bottom.

A triangle is a polygon.

A clown's hat is like a triangle.

25. Ms. Donaldson's class was working on an assignment where they had to find the measures of unknown angles in triangles. One student consistently found the measures of unknown angles in right triangles by subtracting the known angle from 90. For example:



Ms. Donaldson was concerned that this student might run into difficulty when trying to find the measures of unknown angles in more general triangles. Which of the following questions would be best to ask the student in order to help clarify this issue? (Circle ONE answer.)

- a) "What do you get when you add  $90 + 62 + 28$ ?"
- b) "Why does subtracting 62 from 90 give you the measure of the unknown angle?"
- c) "How could you find the missing angle in an isosceles triangle?"
- d) "How did you know that this was a right triangle?"
- e) "What if this angle measured  $17^\circ$  instead of  $62^\circ$ ?"

26. As an early introduction to mathematical proof, Ms. Cobb wants to engage her students in deductive reasoning. She wants to use an activity about the sum of the angles of a triangle, but her students have not yet learned the alternate interior angle theorem. They do, however, know that a right angle is 90 degrees and that a point is surrounded by 360 degrees. Which of the following activities would best fit her purpose? (Circle ONE answer.)

- a) Have students draw a triangle and a line parallel to its base through the opposite vertex. From there, have them reason about the angles of the triangle and the angles the triangle makes with the parallel line.
- b) Have the students use rectangles with diagonals to reason about the sum of the acute angles in a right triangle.
- c) Have students use protractors to measure the angles in several different triangles and from there reason about the sum of the angles of a triangle.
- d) Have students cut out a triangle then tear off the three corners and assemble them, and from there reason about the sum of the angles of a triangle.

27. Mrs. Davies' class has learned how to tessellate the plane with any triangle. She knows that students often have a hard time seeing that any quadrilateral can tessellate the plane as well. She wants to plan a lesson that will help her students develop intuitions for how to tessellate the plane with any quadrilateral.

Which of the following activities would best serve her purpose? (Circle ONE answer.)

- a) Have students cut along the diagonal of various quadrilaterals to show that each can be broken into two triangles, which students know will tessellate.
- b) Provide students with multiple copies of a non-convex kite and have them explore which transformations lead to a tessellation of the plane.
- c) Provide students with pattern blocks so that they can explore which of the pattern block shapes tessellate the plane.
- d) These activities would serve her purpose equally well.

28. Mr. Shephard is using his textbook to plan a lesson on converting fractions to decimals by finding an equivalent fraction. The textbook provides the following two examples:

$$\text{Convert } \frac{2}{5} \text{ to a decimal: } \frac{2}{5} = \frac{4}{10} = 0.4$$

$$\text{Convert } \frac{23}{50} \text{ to a decimal: } \frac{23}{50} = \frac{46}{100} = 0.46$$

Mr. Shephard wants to have some other examples ready in case his students need additional practice in using this method. Which of the following lists of examples would be best to use for this purpose? (Circle ONE answer.)

a)  $\frac{1}{4}$     $\frac{8}{16}$     $\frac{8}{20}$     $\frac{4}{5}$     $\frac{1}{2}$

b)  $\frac{1}{20}$     $\frac{7}{8}$     $\frac{12}{15}$     $\frac{3}{40}$     $\frac{5}{16}$

c)  $\frac{3}{4}$     $\frac{2}{3}$     $\frac{7}{20}$     $\frac{2}{7}$     $\frac{11}{30}$

- d) All of the lists would work equally well.

## MIDDLE SCHOOL CONTENT KNOWLEDGE ITEMS

29. Students sometimes remember only part of a rule. They might say, for instance, "two negatives make a positive." For each operation listed, decide whether the statement "two negatives make a positive" sometimes works, always works, or never works. (Mark SOMETIMES, ALWAYS, NEVER, or I'M NOT SURE)

	Sometimes works	Always works	Never works	I'm not sure
a) Addition	1	2	3	4
b) Subtraction	1	2	3	4
c) Multiplication	1	2	3	4
d) Division	1	2	3	4



30. Mr. Garrison's students were comparing different rectangles and decided to find the ratio of height to width. They wondered, though, if it would matter whether they measured the rectangles using inches or measured the rectangles using centimeters.

As the class discussed the issue, Mr. Garrison decided to give them other examples to consider. For each situation below, decide whether it is an example for which different ways of measuring produce the same ratio or a different ratio. (Circle PRODUCES SAME RATIO, PRODUCES DIFFERENT RATIO, or I'M NOT SURE for each.)

	Produces same ratio	Produces different ratio	I'm not sure
a) The ratio of two people's heights, measured in (1) feet, or (2) meters.	1	2	3
b) The noontime temperatures yesterday and today, measured in (1) Fahrenheit, or (2) Centigrade.	1	2	3
c) The speeds of two airplanes, measured in (1) feet per second, or (2) miles per hour.	1	2	3
d) The growths of two bank accounts, measured in (1) annual percentage increase, or (2) end-of-year balance minus beginning-of-year balance.	1	2	3

31. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e.,  $a(b + c) = ab + ac$ ]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

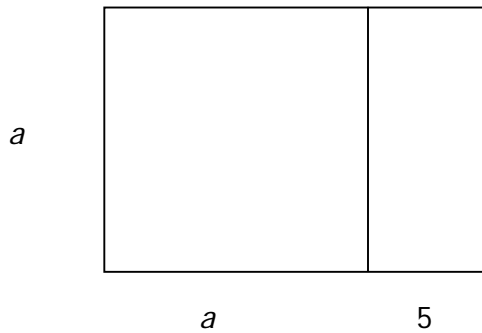
	Applies	Does not apply	I'm not sure
a) Adding $\frac{3}{4} + \frac{5}{4}$	1	2	3
b) Solving $2x - 5 = 8$ for $x$	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding $34 + 25$ using this method: $\begin{array}{r} 34 \\ +25 \\ \hline 59 \end{array}$	1	2	3

32. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions  $a - (b + c)$  and  $a - b - c$  are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes closest to explaining why  $a - (b + c)$  and  $a - b - c$  are equivalent? (Mark ONE answer.)

- a) They're the same because we know that  $a - (b + c)$  doesn't equal  $a - b + c$ , so it must equal  $a - b - c$ .
- b) They're equivalent because if you substitute in numbers, like  $a=10$ ,  $b=2$ , and  $c=5$ , then you get 3 for both expressions.
- c) They're equal because of the associative property. We know that  $a - (b + c)$  equals  $(a - b) - c$  which equals  $a - b - c$ .
- d) They're equivalent because what you do to one side you must always do to the other.
- e) They're the same because of the distributive property. Multiplying  $(b + c)$  by  $-1$  produces  $-b - c$ .

33. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)



	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3
e) $2a + 5$	1	2	3
f) $4a + 10$	1	2	3

34. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$$-x < 9$$

Marcie solved this problem by reversing the inequality sign when dividing by  $-1$ , so that  $x > -9$ . Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the best explanation of why this method works? (Mark ONE answer.)

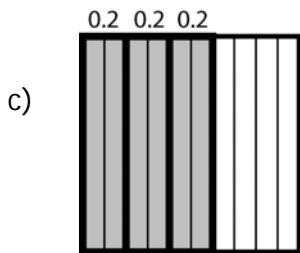
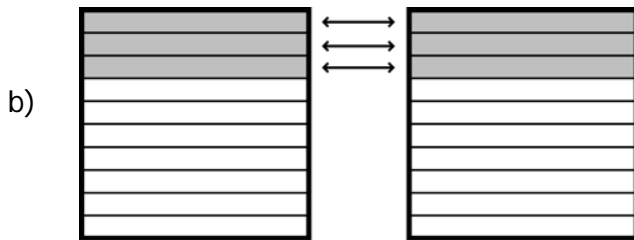
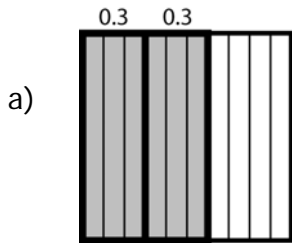
- a) Because the opposite of  $x$  is less than 9.
- b) Because to solve this, you add a positive  $x$  to both sides of the inequality.
- c) Because  $-x < 9$  cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.
- d) Because this method is a shortcut for moving both the  $x$  and 9 across the inequality. This gives the same answer as Marcie's, but in different form:  $-9 < x$ .

35. Ms. Austen was planning a lesson on decimal multiplication. She wanted to connect multiplication of decimals to her students' understanding of multiplication as repeated addition. She planned on reviewing the following definition with her class:

*The repeated addition interpretation of multiplication defines  $a \times b$  as  $b$  added together  $a$  times, or  $a$  groups of  $b$ .*

After reviewing this definition of repeated addition, she planned to ask her students to represent the problem  $0.3 \times 2$  using the repeated addition interpretation of multiplication.

Which of the following representations best illustrates the repeated addition definition of  $0.3 \times 2$ ? (Circle ONE answer.)



- d) These representations illustrate the repeated addition definition of  $0.3 \times 2$  equally well.
- e) Multiplication of decimals cannot be represented using a repeated addition interpretation of multiplication.