

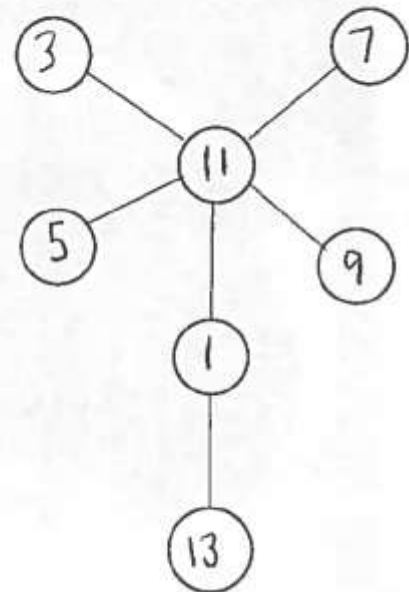
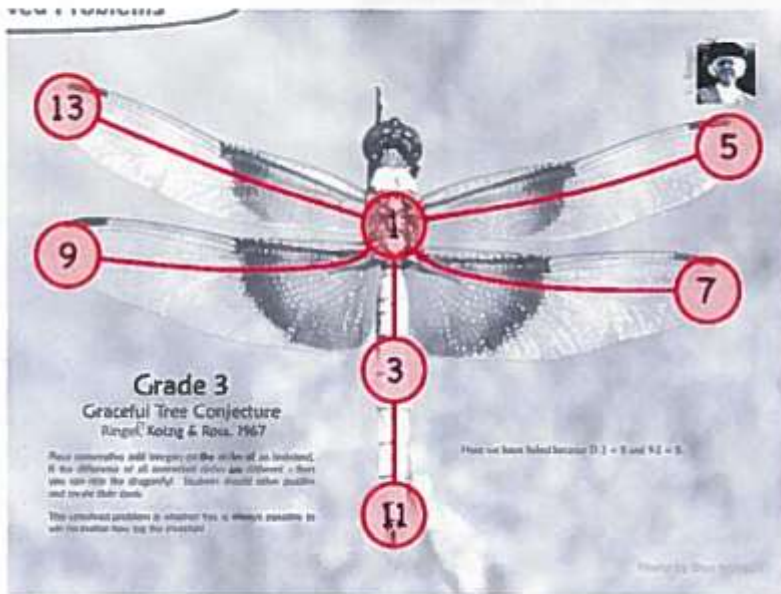
## Graceful Tree Problems

**Goal:** To place consecutive odd numbers in the circles in such a way that the connectors have all the consecutive even numbers. Each tree has all the circles connected with no loops. Each connector has the result from subtracting its adjacent circle numbers. The tree is graceful if it can be solved. That all trees can be solved is a conjecture that has never been proven (and would be worth a lot if you can do it). This is a classic set of problems in graph theory/combinatorics (Cahit, 1990; Fang, 2020; Hartsfield & Ringel, 2013).

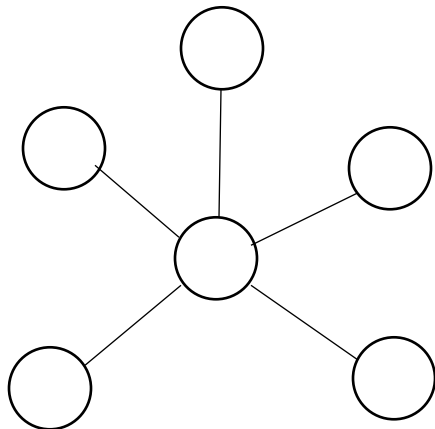
The source and inspiration of many of these problem is the following Youtube video:

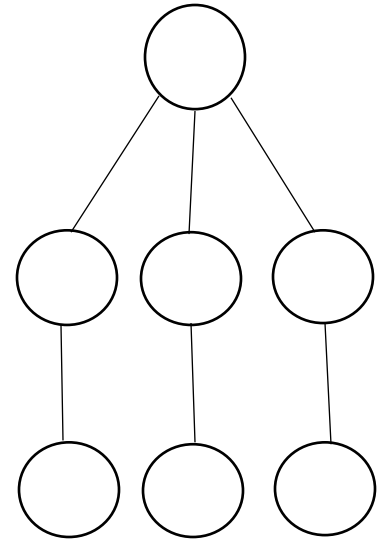
<https://www.youtube.com/watch?v=HJ-MotpqFPY&t=79s>

This is an incorrect dragonfly problem solution since the connecting 8s repeat and there is no 10 on the connectors. Complete the "ungraceful" and the "graceful tree" by including the even numbers on the connectors (2, 4, 6, 8, 10, and 12).



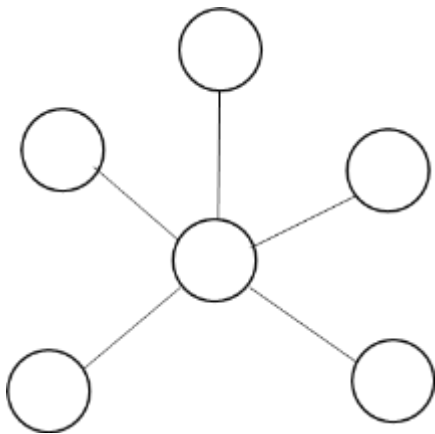
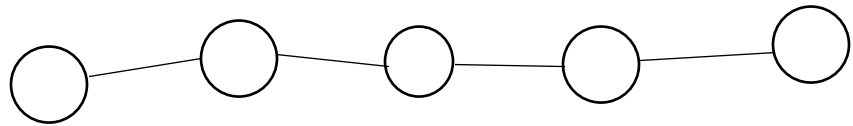
Similar to the dragonfly solution above, solve the butterfly problem with the numbers 1-11.





Solve this crab problem using 1-13 in the circles and connectors.

Solve this snake (or caterpillar) problem using 1-9 in the circles and connectors.



Solve the starfish problem using the numbers 1-11 in the circles and connectors.

