**Math 3001**

**Exam II Practice Test Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

***Chapter 13: Algebraic Thinking: Generalizations, Patterns, and Functions***

**Part I ─
TRUE/FALSE. Circle “T” if the statement is true and “F” if the statement is false.**

1. Although algebra has few connections to real-life applications, it is highly intertwined with other areas of mathematics, and, therefore, is an important topic.
2. Students should begin developing their ability to reason algebraically in elementary school.

1. Children typically have no difficulty understanding the meaning of the = symbol.

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

1. Of the options below, the earliest form of algebraic thinking demonstrated by a student would most likely be
2. Graphing linear functions
3. Recognizing and generalizing patterns
4. Working with expressions that involve variables
5. Making conjectures about number properties
6. Which of the following is NOT one of the three forms of algebraic thinking described by Kaput?
7. Meaningful use of symbols
8. Generalization from arithmetic and from patterns in all of mathematics
9. Number classification
10. Study of patterns and functions

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

1. The ☐ in an open sentence plays the same role as a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, which frequently takes the form of a letter.

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question**.

1. Growing patterns
2. Are frequently functions.
3. Are best represented by numbers only.
4. Should not include fractions and decimals.
5. Have either a recursive formula *or* an explicit formula.

**TRUE/FALSE. Circle “T” if the statement is true and “F” if the statement is false.**
8) Students are more likely to describe the explicit formula for a function verbally before they will be able to describe it symbolically.

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

1. The rate of change of a linear function can be determined from a graph by examining the line’s \_\_\_\_\_\_\_\_\_.
2. Functions can be represented in five ways: context, table, verbal description, symbolic equation, or\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 **MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

1. When students create graphs of functions
2. They are representing them in the manner that makes it the hardest to visualize relationships between patterns.
3. They should place the independent variable (step number) along the vertical axis.
4. It is helpful to provide them with examples within a real-life context.
5. They should always be given specific data, equations, or numbers.
6. A proportional relationship
7. Can only be represented with an equation.
8. Will always have a graph that forms a straight line and passes through the origin.
9. Will always have a positive slope.
10. Is much more challenging for students to generalize than a non-proportional one.

**ESSAY. Write your answer in the space provided or on a separate sheet of paper.**

1. Describe two different ways you could determine whether a function is linear. Describe how these two methods relate to one another, and a possible classroom activity that would help students to see this connection.
2. Describe three different ways algebra can be connected to other areas of the mathematics curriculum.

**Part II ─**

1. Give the 23rd and the *n*th entries for these lists, assuming the patterns continue.

 (a) 1, 3, 9, 27, 81, … (f) 4, 9, 16, 25, 36, . . .

 (b) 5, 7, 9, 11, 13, … (g) 1, 3, 6, 10, 15, . . .

 (c) 2, 4, 5, 7, 8, 10, … (h) 2, 6, 12, 20, 30, . . .

 (d) , … (i) A, C, A, C, A, C, . . .

 (e) 2, 5, 10, 17, 26, … (j) 0, 2, 5, 9, 14, …

2. An **application** involving linear functions involves the relationship between Celsius and Fahrenheit temperature measurements. A common formula (in slope-intercept form) is
F = 1.8C + 32 .

(a) The slope of this line is 1.8 and the y-intercept is 32 since 0˚C = \_\_\_\_˚F.

(b) Write the linear formula relating distance (d) and time (t) for a fixed rate of speed,

 r = 69 mph. d = \_\_\_\_\_\_\_\_\_\_\_\_\_

(c) Another common application involves cost structures. For example, an automobile
 mechanic may charge $238 for parts and $60 an hour for labor. Write the
 corresponding formula, with C for cost and t for time in hours. C = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. A T-shirt company has regular expenses of $5,000 per week in addition to a silk- screening expense of $1.25 per shirt. The company takes in $500 a week from investments they have made. They sell each T-shirt for $5.00 wholesale.

(a) Find the “breakeven point”, the number of T-shirts where profit is 0 (or expenses
 equals income).

(b) If this company sold 1,300 T-shirts, would there be a profit or a loss? How much?

 (b) If the company sold 1,075 T-shirts, figure out their profit or loss.

4. You and your roommates want to subscribe to a movie-rental service. You have two choices:

 Movies-R-Us charges $12/month, plus $4 per movie rented, See-It-Now charges $9/month, plus $5 per movie rented.

 (a) For each choice, write an equation giving the cost *C* for renting *n* movies a month.

 Movies-R-Us: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ See-It-Now: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 (b) Knowing that you would probably average renting 4 movies each month, which
 company would you choose? Explain your reasoning.

5. Make a distance-time and a speed-time graph for the following story. Let the distance refer to the distance from the cave. Use very similar scaling on the time axes.

 “Wile E. is in his cave and then walks slowly toward a canyon, planning to make a trap for Roadrunner. Halfway there, he stops for a short rest. Then, he jogs on to make up for lost time. When he gets to the canyon, he realizes that it is almost time for Animal Planet on TV, so he runs as fast as he can back to the cave.”

6. Make a distance-time and a speed-time graph for the following story. Let the distance refer to the distance from the cave. Use very similar scaling on the time axes.

 You walk for 5 minutes at 1 mile per hour, stop for 2 minutes to watch a nest of baby birds, and then walk for 10 minutes at 2 mph.

For #7-11, sketch a graph to display the relationship between the variables. Label the axes, using the vertical axis for the dependent variable. State any assumptions you make.

1. Prices are now rising more slowly than at

 any time during the last 2 years.

1. The temperature of hot apple cider as it

cools from 85 °C to a room temperature of

 20 °C

1. The speed of a roller coaster over the entire ride

1. No profit will be made on a free

magazine, or on a magazine that is

too expensive for anyone to buy.

In between these extremes lies the

best price for the magazine. (price/profit)

1. When I started to learn the guitar,

I initially made very rapid progress.

But I have found that the better you get,

the more difficult it is to improve still further.

(proficiency/amount of practice)

12. Farmer Rick expects profit on cotton this year to be $20 per bale and profit on peanuts to be $0.15 per pound. He also knows that his farm can average 2 bales of cotton per acre and 600 pounds of peanuts per acre in a very good year.

1. Write a model (formula) for calculating the total profit P from both crops. Use C for

the number of acres of cotton planted and N for the number of acres of peanuts planted.

 (b) Find the total profit if he plants 40 acres of cotton and 75 acres of peanuts.



13. The height in feet, H(t), of a projectile can be approximated over time t (in seconds) using the following function:

(a)&(b) Complete the table for an initial velocity of 64 fps. Show your
 set up on one problem.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Time (seconds) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| Height (feet) |  |  |  |  |  |  |  |  |  |

(c) Sketch the points on the graph provided below, and sketch a curve that contains the

points in your scatter plot. Put the dependent variable on the horizontal axis. Clearly label the axes, and use a consistent scale on each axis.



(d) According to the equation, when does the projectile reach its maximum height? Find
 the maximum height.

14. Imagine that you have a rectangular piece of cardboard that measures 24 inches by 30 inches. You are to form an open box by cutting 4 congruent squares from the corners and folding up the 4 flaps. If you cut a square that measures x in. by x in.,

 (a) Find the general formula for the volume, V, of the box.



 V(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(b) Complete the table below.

 Show your setup on each problem.

|  |  |  |
| --- | --- | --- |
| Size of square (x” by x”) |  | Volume of box (cubic inches) |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |

(c) The volume of the box clearly depends on the size of the square. Using values from

the table above, sketch the points on the grid provided and a curve that contains the points. Put the dependent variable on the vertical axis. Label the axes, and use a consistent scale on each axis.

(d) According to your chart, which size square maximizes the volume of the box?

 (e) According to your chart, what is this maximum volume?

(f) What are the dimensions of the box with greatest volume?

15. Refer to the following broken line graph.



(a) Describe in a sentence the general trend in the graph.

 (b) Choose an appropriate range for when the record of 200 mph was broken.

 1900-1910 1910-1920 1920-1930 1930-1940

 (c) Choose an appropriate range for when the record of 600 mph was broken.

 1950-1960 1960-1970 1970-1980 1980-1990

 (d) According to the graph, has the sound barrier (742 mph) been broken yet?

16. The given scatter plot shows year number on the horizontal axis (x) and U.S. women’s life expectancy in years on the vertical axis (y). [Source: Statistical Abstract of the United States]



Use the equation of the line of best fit to predict what the life expectancy in the U.S. was for women in the year 2020.

**Do your best! Rise to the challenge! Live and learn!**