****PATTERNS, PATTERNS, EVERYWHERE!

## “Algebraic thinking includes recognizing and analyzing patterns, studying and representing relationships, making generalizations, and analyzing how things change.” From *A Journey in Algebraic Thinking* by Cathy Seeley, NCTM President 2004-2006

Some of the number patterns you may encounter are famous in the history of mathematics and critical for the study of algebra. A few of the more important sets of numbers include:

(1) **Fibonacci numbers** — 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . (named after an Italian mathematician
 named Leonardo of Pisa who lived from 1170-c.1250)

 See if you can recognize the pattern above, and write the next few terms in this sequence.

(2) **Triangular numbers** — 1, 3, 6, 10, 15, 21, . . .

 Do these dot patterns make clear the reason for the name?

 Name two popular sports that use these patterns.

(3) **Rectangular numbers** — 2, 6, 12, 20, 30, . . .

 Draw diagrams to illustrate geometrically this sequence.

 Can you develop a formula which works for the nth diagram? Hint: the first drawing has 1×2 dots; the second has 2×3 dots; the third has 3×4 dots, and so on.



(4) **Pascal’s Triangle** —

 (Named for Blaise Pascal, a French physicist and mathematician who lived from 1623-1662, this formation has many interesting patterns and several applications in probability and statistics.)

 See if you can complete the next few rows, and also look

 for some of the patterns in this amazing formation.

(5) **Squares** — 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, . . . . The nth term is n2.

(6) **Cubes** — 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, . . . The nth term is n3.

(7) **Arithmetic sequences**: Add (or subtract) the same number to get from one term to the next.

 For instance, in the arithmetic sequence 2, 5, 8, 11, 14, 17, . . . , we add 3 each time.

 The formula for the nth term of this sequence is .

 Where an = the nth or general term

 a1 = the first term

 n = the term number

 d = the common difference (the difference between a term and the previous term)

 Since a1= 2, n = 1, 2, 3, 4, 5, . . ., and d = 3, the sequence above can be rewritten using this formula as

 2, 2 + 1×3, 2 + 2×3, 2 + 3×3, 2 + 4×3, 2 + 5×3, . . .

 Find the next 3 terms in this sequence.

Find the 12th term. Find the 100th term. Find the nth term.

 There is a famous arithmetic **series** (adding arithmetic terms) story related to a problem Karl Friedrich Gauss was given as a young student. In late 18th-century Germany, a teacher gave a class the following arithmetic problem: 1 + 2 + 3 + 4 + . . . + 98 + 99 + 100. The student who first got the answer was to lay his slate on the table. The teacher had barely announced the problem when Karl laid his slate down, and the class worked the problem the rest of the hour. At the end of the period, the teacher noticed Karl’s slate had on it a single number (the correct answer!).

In later years, Karl confessed to recognizing the pattern 1 + 2 + 3 + . . . + 98 + 99 + 100, in which each pairing results in a sum of 101.

Since there are 50 pairs, the total sum (S) is

50(101) or 5,050.

In general, the sum of natural numbers from 1 to n is given by the formula

 1 + 2 + 3 + 4 + . . . + n = . This is known as **Gauss’ Rule**.

Use Gauss’ Rule to find the following sums:

1. 1 + 2 + 3 + 4 + . . . + 56 + 57 Solution:  
2. 1 + 2 + 3 + 4 + . . . + 999 + 1000 Solution: 

 Gauss’ Rule generalizes for any arithmetic sequence as .

(8) **Geometric sequences**: Multiply (or divide) by the same number to get the next term.

 For instance, in the geometric sequence 1, 2, 4, 8, 16, 32, 64, . . . , we multiply by 2 each time.

 The formula for the nth term of this sequence is .

 Where an = the nth or general term

 a1 = the first term

 n = the term number

 r = the common ratio (the ratio of a term and the previous term)

 Since a1= 1, n = 1, 2, 3, 4, 5, . . ., and r = 2, the sequence above can be rewritten using this formula as

 1, 1×2, 1×22, 1×23, 1×24, 1×25, 1×26, . . .

 Find the next 3 terms in this sequence.

Find the 12th term.

 Find the 100th term. The answer will be too large to calculate. Leave it as a power.

 Find the nth term.

**Classwork Exercises:**

Complete the following sequences of numbers. Then describe the pattern you’ve noticed.

1. 8, 8, 8, 8, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

2. 2, 3, 5, 8, 13, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

3. 81, 100, 121, 144, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

4. 1, 8, 27, 64, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

5. 1, 5, 9, 13, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

6. 5, 10, 20, 40, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

7. 600, 300, 150, 75, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

8. 3, -6, 12, -24, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

9. , \_\_\_\_ , \_\_\_\_ , \_\_\_\_

10. 0.6, 0.2, -0.2, -0.6, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

11. \_\_\_\_ , \_\_\_\_ , \_\_\_\_

12. 9, 18, 27, 36, 45, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

13. 7, 14, 21, 28, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

14. 1998, 2002, 2006, 2010, \_\_\_\_ , \_\_\_\_ , \_\_\_\_ (Winter Olympic years)

15. A, D, E, H, \_\_\_\_ , \_\_\_\_ , \_\_\_\_

16. A, B, B, A, A, B, B, \_\_\_\_ , \_\_\_\_ , B, \_\_\_\_ , A (Look up Romans 8:15 if you’d like.)

17. clap, clap, stomp, clap, clap, \_\_\_\_\_\_\_ , clap, \_\_\_\_\_\_\_ , stomp, \_\_\_\_\_\_\_

18. , , , , \_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_

19. , , , , \_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_

\*20. 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, 1 + 2 + 3 + 4 + 5, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ,

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ,

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

 [This relates to the traditional English song “The Twelve Days of Christmas” which starts with the true love giving a partridge in a pear tree on the first day, two turtledoves and a partridge in a pear tree on the second day, three French hens and two turtledoves and a partridge in a pear tree on the third day, and so on.]

 Find the grand total number of gifts over the 12 days. \_\_\_\_\_\_\_ WOW!!