1.   List George Polya's 4 general steps to problem solving. Use complete sentences.

(1) _____

(2) _____

(3) _____

(4) _____

2.   In each of the following, list terms that continue the pattern. Then tell whether the
sequence is arithmetic, geometric, or neither.

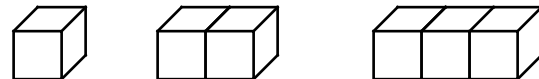(a) 0, 20, 40, 60, 80, _____ , _____ , _____   _____

(b) 5, –50, 500, –5000, 50000, _____ , _____ , _____   _____

(c) 1, 4, 9, 16, 25, _____ , _____ , _____   _____

(d) 7, 8, 15, 23, 38, _____ , _____ , _____   _____

3.   In the following sequence, the figures are made of cubes that are glued together. If the
exposed surfaces need to be painted, how many squares will be painted in the
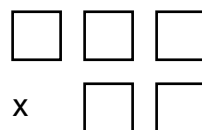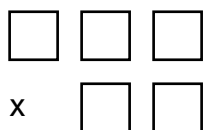
(a) 5th figure?

(b) n^{th} figure?

4.   Place the digits 2, 3, 5, 7, and 8 in the following boxes for the

(a) greatest product   (b) least product

□ □ □   □ □ □

x □ □   x □ □
_____   _____

5. In a rectangular field, the length is twice the width. If the perimeter is 300 yards, find the area.

6. If it takes 12 minutes to cut a log into 3 pieces, how long will it take to cut the log into 9 pieces? (Hint: How long does a single cut take?)

7. How many ways can you make change for a $50 bill using $5, $10, and $20 bills? Show them in the table below.

$5 bills													
$10 bills													
$20 bills													

8. Molly and Samantha began saving money at the same time. Molly plans to save $4 a month, and Samantha plans to save $6 a month. After how many months will Samantha have exactly $10 more than Molly?

9. Place the digits 2, 4, 5, 6, and 9 in the boxes for the least sum.

    ☐ ☐ ☐
    + ☐ ☐
    ‾‾‾‾‾‾‾

10. Place the digits 1, 4, 5, 7, and 8 in the boxes for the greatest sum.

    ☐ ☐ ☐
    + ☐ ☐
    ‾‾‾‾‾‾‾

11. Find the 20th term in the following sequences:

(a) 5, 7, 9, 11, . . .

(b) 2, 6, 18, 54 . . .

(c) 1, –1, 1, –1, . . .

(d) 5, 10, 20, 40, . . .

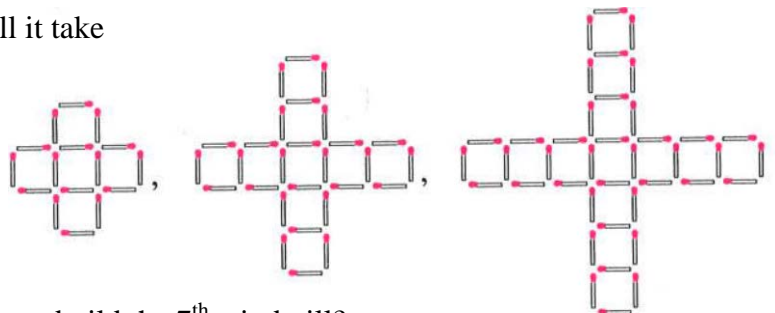12. Find the indicated sums.

(a) 1 + 5 + 9 + 13 + 17 . . . + 97 + 101

(b) the first 12 terms of 18 + 16 + 14 + . . .

(c) 15 + 17 + 19 + 21 + . . . + 247 + 249

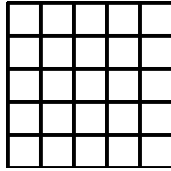(d) the first 50 terms of 7 + 10 + 13 + . . .

13. The first "windmill" pattern below takes 5 matchstick squares to build, the second takes 9 squares to build, and the third takes 13 squares to build.

(a) How many matchstick squares will it take
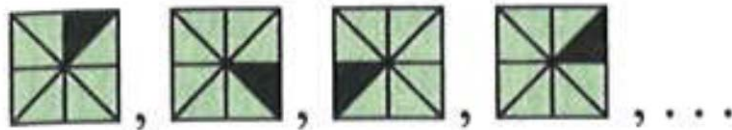to build the 7th windmill?

(b) How many matchsticks will it take to build the 7th windmill?

14.   How many squares are there in the following diagram?

15.   If fence posts are to be placed in a row 3 meters apart (center-to-center), how many posts are needed for 30 meters of fence?

16.   Determine a possible pattern, and draw the next figure according to that pattern.

17.   Determine the following sums:

(a) $1 + 2 + 3 + 4 + 5 + \ldots + 999 + 1{,}000$ (b) $1 + 4 + 7 + 10 + \ldots + 298$

18.   In each of the following, list terms that continue the pattern. Then give the name of the sequence (arithmetic, geometric, Fibonacci, rectangular, triangular, cubes, squares, etc.).

(a) 1, 1, 2, 3, 5, 8, 13, 21, _____ , _____   _____

(b) 1, 8, 27, 64, 125, 216, _____ , _____   _____

(c) 2, 6, 12, 20, 30, 42, _____ , _____   _____

(d) 1, 3, 6, 10, 15, 21, _____ , _____   _____

19. Make up your own patterns/problem solving problem, and then show your reasoning toward the solution. Be creative!

20. In the following patterns, identify the core and the 20th term in the sequence.

(a) 1, 1, 2, 1, 1, 2, 1, 1, 2, …   Core: _____ 20th term: _____

(b) A, A, B, B, A, A, B, B, …   Core: _____ 20th term: _____

21. Rewrite $1 \times 10^5 + 3 \times 10^3 + 8 \times 10^2 + 4 \times 10^0$ as a base-ten numeral.

22. Write the negation of each of the following statements.

(a) $3 \cdot 5 = 15$

(b) All whales are mammals.

(c) For some natural numbers n, $2 < n < 6$.

(d) No dogs have fleas.

23. Complete the following truth table for the various types of propositions.

(a)   Negation

p	~ p
T	
F	

(b)   And statements (Conjunctions)

p	q	p and q
T	T	
T	F	
F	T	
F	F	

23. (c) Or statements (Disjunctions)

p	q	p or q
T	T	
T	F	
F	T	
F	F	

(d) If . . . then statements (Conditionals)

p	q	if p, then q
T	T	
T	F	
F	T	
F	F	

24. Write the general form for each variation of the conditional statement, along with the example (using the given conditional statement).

Conditional: If p, then q.   If you build it, then they will come.

Converse: _____   _____

Inverse: _____   _____

Contrapositive: _____   _____

25. Decide on the truth value (True or False) of each of the following statements.

(a) $2 + 3 = 5$ and $9 \times 6 = 56$

(b) A set with n elements has 2^n subsets.

(c) If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

(d) $3 \cdot 4 = 7$ or $-4 + -5 = -9$

26. For each argument, form a conclusion that follows logically from the given statements.

(a) If I study for the tests, I will pass the tests.
   If I pass the tests, then I will pass the course.
   If I pass the course, then I will graduate.

26. (b) If a student is a freshman, then the student takes FIRE 1000.
         Jackie is a freshman.

27. If q stands for "You said goodbye" and r stands for "I said hello," write each statement in symbolic form.

     (a) You said goodbye, and I didn't say hello.

     (b) I said hello, or you did not say goodbye.

     (c) It is false that both you said goodbye and I said hello.

28. (a) Determine whether the given form represents a valid argument._____

     $\sim p$
     $q \rightarrow p$
     $\underline{(p \vee q) \rightarrow r}$
     $\therefore r$

     (b) Use a truth table to determine whether the given argument is valid. _____

     If you pay more for your phone plan, then you will have more calling minutes.
     If you pay more for your phone plan or have more calling minutes, then you will call your mother more often.
     You paid more for your phone plan.
     Therefore, you will call your mother more often.

29. Fill in the blank with the symbol ($\in$, $\notin$, $\subset$, or $\not\subset$) that makes the following statement true:
                400 _____ $\{x \mid x = 2^n \text{ and } n \in N\}$

30. Fill in the blank with the symbol $(\in \text{ or } \notin)$ which makes the statement true:

                55 _____ $\{x \mid x = 2n + 1 \text{ and } n \in N\}$

31. Rewrite the following using mathematical symbols:
    The set consisting of the elements 1 and 2 is a proper subset of {1, 2, 3, 4, 5, 6, 7}.

32. Fill in the blank with the correct set symbol $(\subset, \subseteq, \in, \text{ or } \notin)$ if
    A = {1, 3, 5} and B = {1, 2, 3, 4, 5, 6, 135}.

    A _____ B

33. Let N = {1, 2, 3, 4, 5, ... } be the universal set, and let A = {1, 3, 5, 7, 9, ... },
    B = {1, 4, 9, 16, 25, ... }, and C = {3, 6, 9, 12, 15, ... }. List at least 3 elements in

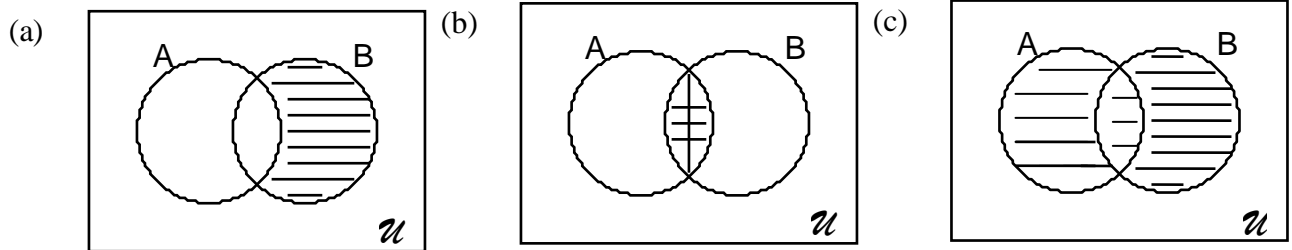   (a) A ∩ B   (b) A ∪ C   (c) $\overline{C}$ (d) $\overline{A \cup B}$

34. Use set notation to identify each of the following shaded regions.

    (a)
    (b)
    (c)

35. In a club with 50 members, 23 take mathematics (and love it!), 9 take both mathematics
    and biology, and 3 take neither mathematics nor biology. How many take biology but not
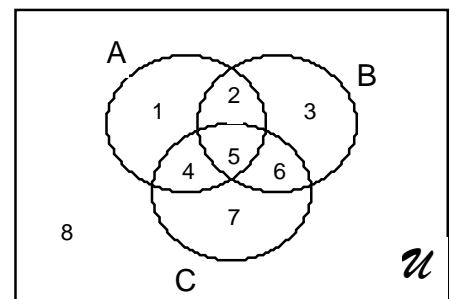   mathematics? (Hint: Draw a Venn diagram.)

36. List (by number) the region(s) corresponding to

    (a) $\left(\overline{A \cap B}\right) \cap C$ (b) C − A

    (c) $\left(A \cup \overline{C}\right) \cap \overline{B}$ (d) $\overline{B - C}$

37. How many correspondences are there between the two sets {Andy, Fred, Mandy, Paul, Sally, Tamara} and {1, 2, 3, 4, 5, 6}? (Hint: Think of how many different ways to place these 6 swimmers in 6 pool lanes in a swim tournament.]

38. How many subsets are there of the set $\{O, \Delta, \perp, \Diamond\}$? Pascal's triangle may help.

39. A new model of car is available in 3 exterior colors and 4 interior colors. How many color schemes are available for this car? (Hint: Use a tree diagram with specific colors.)

40. If A = {1, 2, 3, 4} and B = {a, b, c},

   (a) Find A × B.

   (b) Fill in the blanks.

   n(A) = _____   n(B) = _____   n(A × B) = _____

41. Make up your own sets/set operations/bases/numeration systems problem, and then show your reasoning toward the solution. Be creative!

Do your best! Rise to the challenge! Live and learn!