
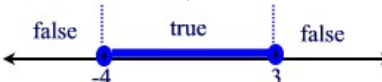


# Algebra 2 – Things to Remember!



<p><b>Exponents:</b></p> $x^0 = 1$ $x^m \cdot x^n = x^{m+n}$ $\frac{x^m}{x^n} = x^{m-n}$ $(xy)^n = x^n \cdot y^n$ $x^{-m} = \frac{1}{x^m}$ $(x^n)^m = x^{n \cdot m}$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	<p><b>Complex Numbers:</b></p> $\sqrt{-1} = i \quad \sqrt{-a} = i\sqrt{a}; a \geq 0$ $i^2 = -1 \quad i^4 = i^2 = -1$ <p>divide exponent by 4, use remainder, solve.</p> <p><math>(a + bi)</math> conjugate <math>(a - bi)</math></p> $(a + bi)(a - bi) = a^2 + b^2$ $ a + bi  = \sqrt{a^2 + b^2}$ absolute value=magnitude	<p><b>Logarithms</b></p> $y = \log_b x \Leftrightarrow x = b^y$ <p><math>\ln x = \log_e x</math> natural log  <math>e = 2.71828\dots</math>  <math>\log x = \log_{10} x</math> common log</p> <p>Change of base formula:  <math display="block">\log_b a = \frac{\log a}{\log b}</math></p> <p><b>Properties of Logs:</b></p> $\log_b b = 1 \quad \log_b 1 = 0$ $\log_b (m \cdot n) = \log_b m + \log_b n$ $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$ $\log_b (m^r) = r \log_b m$ <p>Domain: <math>\log_b x</math> is <math>x &gt; 0</math></p>
<p><b>Factoring:</b></p> <p>Look to see if there is a GCF (greatest common factor) first. <math>ab + ac = a(b + c)</math></p> $x^2 - a^2 = (x - a)(x + a)$ $(x + a)^2 = x^2 + 2ax + a^2$ $(x - a)^2 = x^2 - 2ax + a^2$ <p><b>Factor by Grouping:</b></p>  $x^3 + 2x^2 - 3x - 6$ <p><math>(x^3 + 2x^2) - (3x + 6)</math> group</p> <p><math>x^2(x + 2) - 3(x + 2)</math> factor each</p> <p><math>(x^2 - 3)(x + 2)</math> factor</p>	<p><b>Exponentials</b> <math>e^x = \exp(x)</math></p> $b^x = b^y \rightarrow x = y \quad (b > 0 \text{ and } b \neq 1)$ <p>If the bases are the same, set the exponents equal and solve.</p> <p><b>Solving exponential equations:</b></p> <ol style="list-style-type: none"> <li>1. Isolate exponential expression.</li> <li>2. Take <math>\log</math> or <math>\ln</math> of both sides.</li> <li>3. Solve for the variable.</li> </ol> <p><math>\ln(x)</math> and <math>e^x</math> are inverse functions</p> $\ln e^x = x \quad e^{\ln x} = x$ $\ln e = 1 \quad e^{\ln 4} = 4$ $e^{2 \ln 3} = e^{\ln 3^2} = 9$	<p><b>Quadratic Equations:</b> <math>ax^2 + bx + c = 0</math> (Set = 0.)</p> <p>Solve by factoring, completing the square, quadratic formula.</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p><math>b^2 - 4ac &gt; 0</math> two real unequal roots</p> <p><math>b^2 - 4ac = 0</math> repeated real roots</p> <p><math>b^2 - 4ac &lt; 0</math> two complex roots</p> <p>Square root property: If <math>x^2 = m</math>, then <math>x = \pm\sqrt{m}</math></p> <p><b>Completing the square:</b> <math>x^2 - 2x - 5 = 0</math></p> <ol style="list-style-type: none"> <li>1. If other than one, divide by coefficient of <math>x^2</math></li> <li>2. Move constant term to other side <math>x^2 - 2x = 5</math></li> <li>3. Take half of coefficient of <math>x</math>, square it, add to both sides</li> </ol> $x^2 - 2x + \boxed{1} = 5 + \boxed{1}$ <ol style="list-style-type: none"> <li>4. Factor perfect square on left side. <math>(x - 1)^2 = 6</math></li> <li>5. Use square root property to solve and get two answers. <math>x = 1 \pm \sqrt{6}</math></li> </ol>
<p><b>Variation:</b> always involves the constant of proportionality, <math>k</math>. Find <math>k</math>, and then proceed.</p> <p><b>Direct variation:</b> <math>y = kx</math></p> <p><b>Inverse variation:</b> <math>y = \frac{k}{x}</math></p> <p>Varies jointly: <math>y = kxj</math></p> <p>Combo: Sales vary directly with advertising and inversely with candy cost.</p> $y = \frac{ka}{c}$	<p><b>Absolute Value:</b> <math> a  &gt; 0</math></p> $ a  = \begin{cases} a; & a \geq 0 \\ -a; & a < 0 \end{cases}$ $ m  = b \Rightarrow m = -b \text{ or } m = b$ $ m  < b \Rightarrow -b < m < b$ $ m  > b \Rightarrow b > m \text{ or } b < m$	<p><b>Sum of roots:</b> <math>r_1 + r_2 = -\frac{b}{a}</math>    <b>Product of roots:</b> <math>r_1 \cdot r_2 = \frac{c}{a}</math></p> <p><b>Inequalities:</b> <math>x^2 + x - 12 \leq 0</math> Change to =, factor, locate critical points on number line, check each section.</p> $(x + 4)(x - 3) = 0$ $x = -4; x = 3$  <p><b>ANSWER:</b> <math>-4 \leq x \leq 3</math> or <math>[-4, 3]</math> (in interval notation)</p>

**Radicals:** Remember to use fractional exponents.

$$\sqrt[n]{x} = x^{\frac{1}{n}} \quad x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{a^n} = a \quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

**Simplify:** look for perfect powers.

$$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$$

$$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \cdot 9x^8xy^8z^3} = 2x^2y^2z\sqrt[3]{9x}$$

**Use conjugates to rationalize denominators:**

$$\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$$

**Equations:** isolate the radical; square both sides to eliminate radical; combine; solve.

$$2x-5\sqrt{x}-3=0 \rightarrow (2x-3)^2 = (5\sqrt{x})^2$$

$$4x^2-12x+9=25x \rightarrow \text{solve: } x=9; x=1/4$$

**CHECK ANSWERS. Answer only  $x = 9$ .**

**Functions:** A function is a set of ordered pairs in which each  $x$ -element has only ONE  $y$ -element associated with it.

**Vertical Line Test:** is this graph a function?

**Domain:**  $x$ -values used; **Range:**  $y$ -values used

**Onto:** all elements in B used.

**1-to-1:** no element in B used more than once.

**Composition:**  $(f \circ g)(x) = f(g(x))$

**Inverse functions  $f$  &  $g$ :**  $f(g(x)) = g(f(x)) = x$

**Horizontal line test:** will inverse be a function?

**Transformations:**

$-f(x)$  over  $x$ -axis;  $f(-x)$  over  $y$ -axis

$f(x+a)$  horizontal shift;  $f(x)+a$  vertical shift

$f(ax)$  stretch horizontal;  $af(x)$  stretch vertical

**Working with Rationals ( Fractions):**

**Simplify:**

remember to look for a factoring of  $-1$ :

$$\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$$

**Add:** Get the common denominator.

Factor first if possible:

**Multiply and Divide:** Factor First

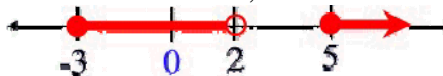
**Rational Inequalities**

$$\frac{x^2-3x-15}{x-2} \geq 0 \text{ The critical values}$$

from factoring the numerator are  $-3, 5$ .

The denominator is zero at  $x = 2$ .

Place on number line, and test sections.



**Sequences**

**Arithmetic:**  $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

**Geometric:**  $a_n = a_1 \cdot r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

**Recursive:** Example:

$$a_1 = 4; \quad a_n = 2a_{n-1}$$

**Binomial Theorem:**

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

**Solving Rational Equations:**

Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by  $2x^2-9x-5$  and get

$$22-3(x-5) = 2(2x+1)$$

$$22-3x+15 = 4x+2$$

$$37-3x = 4x+2$$

$$35 = 7x$$

$$5 = x$$

Great! But the only problem is that

$x = 5$  does not CHECK!!!! There is no solution.

Extraneous root.

**Motto: Always CHECK ANSWERS.**

**Equations of Circles:**  $x^2 + y^2 = r^2$  center origin

$$(x-h)^2 + (y-k)^2 = r^2 \text{ center at } (h,k)$$

$$x^2 + y^2 + Cx + Dy + E = 0 \text{ standard form}$$

**Complex Fractions:**

Remember that the fraction bar means divide:

Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{2-4x}{x^2} \cdot \frac{x^2}{4x-2} = -1$$

Method 2: Mult. all terms by common denominator for all.

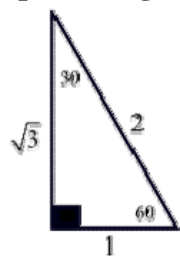
$$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{x^2 \cdot \frac{2}{x^2} - x^2 \cdot \frac{4}{x}}{x^2 \cdot \frac{4}{x} - x^2 \cdot \frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

# Trigonometry – Things to Remember!

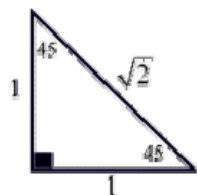


**Arc Length of a Circle** =  $\theta r$  (in radians)

## Special Right Triangles



30°-60°-90° triangle  
side opposite 30° = ½ hypotenuse  
side opposite 60° = ½ hypotenuse  $\sqrt{3}$



45°-45°-90° triangle  
hypotenuse = leg  $\sqrt{2}$   
leg = ½ hypotenuse  $\sqrt{2}$

**Law of Sines:** uses 2 sides and 2 angles  
 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$  Has an ambiguous case.

**Law of Cosines:** uses 3 sides and 1 angle  
 $c^2 = a^2 + b^2 - 2ab \cos C$

**Area of triangle:**  $A = \frac{1}{2} ab \sin C$   
**Area of parallelogram:**  $A = ab \sin C$

**Pythagorean Identities:**  
 $\sin^2 \theta + \cos^2 \theta = 1$      $\tan^2 \theta + 1 = \sec^2 \theta$   
 $1 + \cot^2 \theta = \csc^2 \theta$

## Radians and Degrees

Change to radians multiply by  $\frac{\pi}{180}$

Change to degrees multiply by  $\frac{180}{\pi}$

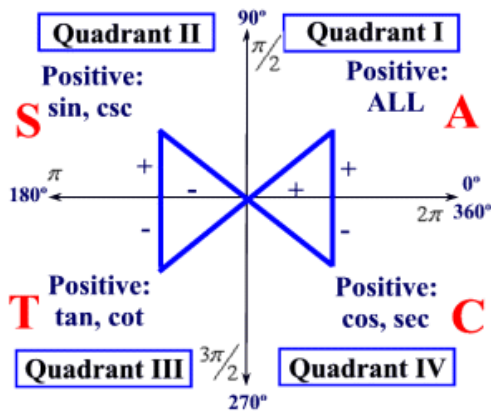
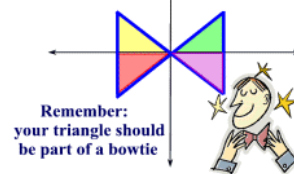
## Quadrantal angles – 0, 90, 180, 270

**CoFunctions:** examples  
 $\sin \theta = \cos(90^\circ - \theta)$ ;  $\tan \theta = \cot(90^\circ - \theta)$

## Inverse notation:

$\arcsin(x) = \sin^{-1}(x)$   
 $\arccos(x) = \cos^{-1}(x)$   
 $\arctan(x) = \tan^{-1}(x)$

Reference triangles are drawn to the x-axis.



## Trig Functions

$\sin \theta = \frac{o}{h}$ ;  $\cos \theta = \frac{a}{h}$ ;  $\tan \theta = \frac{o}{a}$

$\csc \theta = \frac{h}{o}$ ;  $\sec \theta = \frac{h}{a}$ ;  $\cot \theta = \frac{a}{o}$

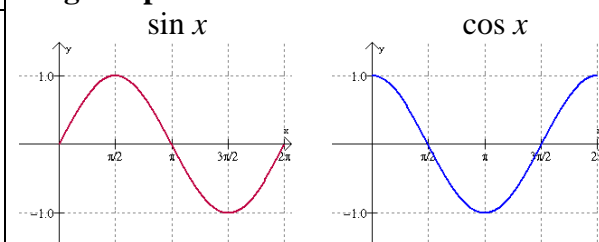
## Reciprocal Functions

$\sin \theta = \frac{1}{\csc \theta}$ ;  $\cos \theta = \frac{1}{\sec \theta}$ ;  $\tan \theta = \frac{1}{\cot \theta}$

$\csc \theta = \frac{1}{\sin \theta}$ ;  $\sec \theta = \frac{1}{\cos \theta}$ ;  $\cot \theta = \frac{1}{\tan \theta}$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$        $\cot \theta = \frac{\cos \theta}{\sin \theta}$

## Trig Graphs



amplitude =  $\frac{1}{2} | \max - \min |$  (think height)

period = horizontal length of 1 complete cycle

frequency = number of cycles in  $2\pi$

sinusoidal curve = any curve expressed as  
 $y = A \sin(B(x - C)) + D$

phase shift = measure of horizontal shifting

# Statistics and Probability – Things to Remember!

Statistics:

$$\text{mean} = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

*median* = middle number in ordered data

*mode* = value occurring most often

*range* = difference between largest and smallest

**mean absolute deviation (MAD):**

$$\text{population MAD} = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$$

**variance:**

$$\text{population variance} = (\sigma x)^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

**standard deviation:**

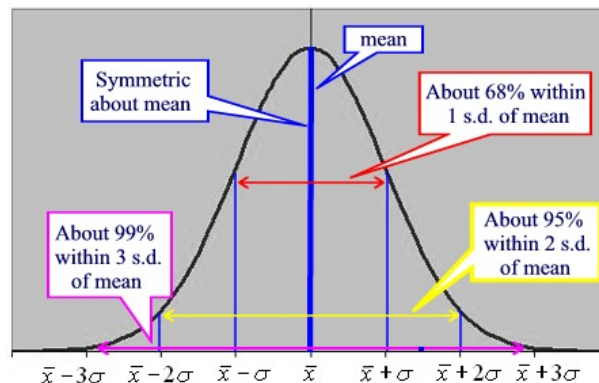
*population* standard deviation =

$$\sigma x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$S_x$  = sample standard deviation

$\sigma_x$  = population standard deviation

## Normal Distribution and Standard Deviation



## Binomial Probability

$${}_n C_r \cdot p^r \cdot q^{n-r} \text{ exactly } r \text{ times}$$

$$\text{or } \binom{n}{r} \cdot p^r \cdot (1-p)^{n-r}$$

When computing "**at least**" and "**at most**" probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
- all probabilities smaller than the given probability ("at most")

## Probability

**Permutation:** without replacement and order matters

$${}_n P_r = \frac{n!}{(n-r)!}$$

**Combination:** without replacement and order does not matter

$${}_n C_r = \binom{n}{r} = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

## Empirical Probability

$$P(E) = \frac{\text{\# of times event } E \text{ occurs}}{\text{total \# of observed occurrences}}$$

## Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{\# of outcomes in } E}{\text{total \# of outcomes in } S}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A') = 1 - P(A)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

for not mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

for mutually exclusive

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \text{ (conditional)}$$