Hypothesis Testing   Pretzel Problem   One-Sample Z-Test

A snack-food company produces a 454-g bag of pretzels. Although the actual net weights deviate slightly from the 454 g and vary from one bag to another, the company insists that the mean net weight be kept at 454 g. If the mean net weight is less than 454 g, the company will be short-changing its customers; and if the mean exceeds 454 g, then the company will be unnecessarily overfilling the bags. As part of its program, the quality-assurance department periodically performs a hypothesis test to decide whether the packaging machine is working properly, that is, to decide whether the mean net weight of all bags packaged is 454 g.

The null and alternative hypotheses for that hypothesis test are

H_0: __________ (the packaging machine is working properly)
H_1: __________ (the packaging machine is not working properly) ,
where μ is the mean weight of all bags or pretzels packaged.

Which of these hypotheses is the company's claim? _______

We will assume the net weights are normally distributed and that the population standard deviation of all such weights is 7.8 g (from previous research). A random sample of 25 bags of pretzels has the net weights show below.

465	456	438	454	446
449	442	449	446	447
468	433	454	463	450
446	447	456	452	444
447	456	456	435	450

The logic behind carrying out a hypothesis test is this: If the null hypothesis is true, then the mean weight of the sample of 25 bags of pretzels should be approximately equal to 454 g. We say "approximately" because we cannot expect a sample mean to exactly equal the population mean due to sampling error. However, if the sample mean differs "too much" from 454 g, then we would be inclined to reject the null hypothesis in favor of the alternative hypothesis.

Find the sample mean $\bar{X}$ for the sample above.

By the central limit theorem, we use the test statistic $z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$

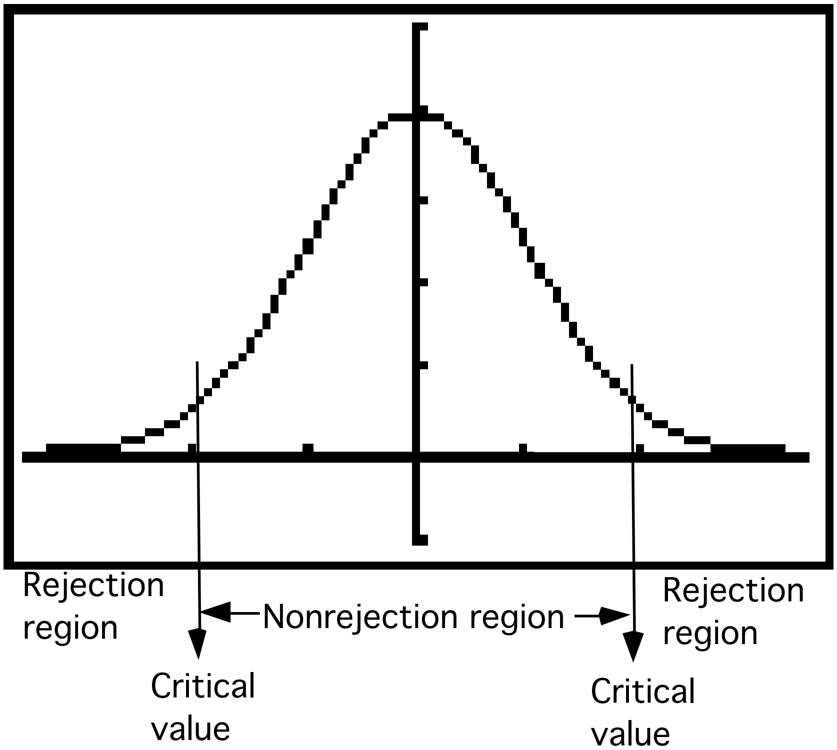
which tells us how many standard deviations the sample mean is from the null hypothesis population mean of 454 g.

Next, we'll decide on a significance level. The significance level is the probability of rejecting a true null hypothesis. Common levels are 0.10, 0.05, and 0.01. We'll choose to use 0.05 or 5%.

Ours is a two-tailed test, and we'll use a picture to show the rejection region, the nonrejection region, and the critical values used in decision-making. The null hypothesis is rejected when the test statistic is either too small or too large.

Whenever we conduct a hypothesis test, it's possible that the decision we reach will be incorrect. This is because partial information, obtained from a sample, is used to draw conclusions about the entire population. Rejecting a true null hypothesis is a "Type I error", and nonrejection of a false null hypothesis is a "Type II error". Again, the probability of a Type I error is given by α.

Find the critical values associated with α = 0.05 and write the areas associated with the area in each "tail".



Compute z (the test statistic).

If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0 (Interpret this as there is not enough evidence to reject H_0.)

Decision: ____________________

State the conclusion in words.

Find the p-value. What does this value say about the strength of the evidence against H_0?