Math 2101 Statistics

Chapter 8 —

1. (a) $H_0: \mu = 24.6$ years (claim) $H_1: \mu \neq 24.6$ years (b) $H_0: \mu \leq 25.4$ years $H_1: \mu > 25.4$ years (claim) (c) $H_0: \mu \geq 88$ points $H_1: \mu < 88$ points (claim) (d) $H_0: \mu \geq 70$ bpm $H_1: \mu < 70$ bpm (claim) (e) $H_0: \mu = 8.2$ lb (claim) $H_1: \mu \neq 8.2$ lb $H_0: \mu \leq \$19,410$

 $H_1: \mu > $19,410$

Claim: H₁

Test statistic:

All hypothesis test calculated values (critical values, test statistics, and p-values) are approximate.

Critical value: 2.33

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{22098 - 19410}{6050 / \sqrt{40}} \approx 2.81$$

Decision: Reject H₀.

Conclusion: There is strong evidence (p-value ≈ 0.0025) in favor of the claim that the cost has increased.

3.
$$\begin{array}{c} H_0: \mu \geq 11.52 \text{ in.} \\ H_1: \mu < 11.52 \text{ in} \end{array}$$
 Claim: H_1

Test statistic:

Critical value:: -1.833

$$t = \frac{X - \mu}{s / \sqrt{n}} = \frac{7.42 - 11.52}{1.3 / \sqrt{10}} \approx -9.97$$

Decision: Reject H₀.

Conclusion: There is extremely strong evidence (p-value ≈ 0.0000018) in favor of the claim that the rainfall is below average.

4. $\begin{array}{l} H_0: p = 64.7\% \\ H_1: p \neq 64.7\% \end{array}$ Claim: H_1

$$p = 0.647, q = 0.353, \hat{p} = \frac{92}{150} = 0.61\overline{3}$$

Test statistic:

Critical value(s): ± 2.575

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{92}{150} - 0.647}{\sqrt{0.647 \cdot 0.353}} \approx -0.863$$

Decision: Do not reject H_0 .

Conclusion: There is very little evidence (p-value = 0.3883) in favor of the claim that there is a difference from the national population on the proportion who own their home.

5. $\begin{array}{l} H_0: \sigma \leq 20 \text{ calories} \\ H_1: \sigma > 20 \text{ calories} \end{array} \quad \text{Claim: } H_1 \end{array}$

Test statistic:

Critical value: 36.191

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19 \cdot 35.109^2}{20^2} \approx 58.550$$

Decision: Reject H_0 .

Conclusion: There is very strong evidence (p-value < 0.005, $p \approx 0.00000655$) in favor of the claim that the standard deviation is greater than 20 calories.

Chapter 9 —

6.
$$(\overline{X_1} - \overline{X_2}) - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X_1} - \overline{X_2}) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(9.2 - 8.8) - 1.96 $\sqrt{\frac{0.3^2}{27} + \frac{0.1^2}{30}} < \mu_1 - \mu_2 < (9.2 - 8.8) + 1.96 \sqrt{\frac{0.3^2}{27} + \frac{0.1^2}{30}}$

 $0.28 v < \mu_1 - \mu_2 < 0.52 v$ Since the 95% confidence interval for the difference in means does not contain 0, if we set up a hypothesis test for these batteries' mean voltage performance with $H_0: \mu_1 = \mu_2$, we would reject the null at $\alpha = 0.05$.

7. $H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \text{ (Claim)}$

Test statistic:

Critical value(s):: ± 2.33

$$z = \frac{\left(\overline{X_1} - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(9.2 - 8.8) - (0)}{\sqrt{\frac{0.3^2}{27} + \frac{0.7^2}{30}}} \approx 6.61$$

Decision: Reject H_0 .

Conclusion: There is extremely strong evidence (p-value<0.0001, p-value \approx 0.000000000397) to support the claim that there is a significant difference in the voltage output of the two battery brands.

8. Step 1— $\begin{array}{l}
H_0: \sigma_1^2 = \sigma_2^2 \\
H_1: \sigma_1^2 \neq \sigma_2^2 \text{ (Claim)}
\end{array}$ Because of the F Test, we are using subscript 1 for IRS tax preparers, and we are using subscript 2 for the volunteer tax preparers.

Test statistic:
$$F = \frac{s_1^2}{s_2^2} = \frac{5.6^2}{4.3^2} = 1.70$$
 Critical value: 4.19

Decision: Do not reject H_0 . Assume that the variances are equal.

Step 2 —
$$H_0: \mu_1 = \mu_2$$

 $H_1: \mu_1 \neq \mu_2$ (Claim)

Critical value(s):: ± 2.508 [df = 10 + 14 - 2 = 22]

 $t = \frac{\left(\overline{X_1} - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\left(21 - 27\right) - \left(0\right)}{\sqrt{\frac{9 \cdot 5.6^2 + 13 \cdot 4.3^2}{22}} \sqrt{\frac{1}{10} + \frac{1}{14}}} \approx -2.973$

Decision: Reject H_0 .

Conclusion: There is strong evidence (p-value < 0.005) in favor of the claim that there is a significant difference in the average times of the two types of tax return preparers.

The 98% confidence interval is -11.1 < $\mu_1 - \mu_2 < -0.93$.

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Twin A (X_1)	87	92	78	83	88	90	84	93
Twin=B $O(X_2)$	D ⁸³ =1	$0495\overline{D}$	$=\frac{199}{8}=1$.2583	86 _D =	$\sqrt{\frac{103-1}{7}}$	0 /80 ≈	3.699
$\mathbf{D} = \mathbf{X}_1 - \mathbf{X}_2$	4	-3	-1	0	2	-3	4	7
D^2	16	9	1	0	4	9	16	49

$$\begin{split} H_0: \mu_D &= 0\\ H_1: \mu_D \neq 0 \ (Claim) \end{split}$$

Test statistic:

Critical value(s):: ± 3.499 [df = 7]

$$t = \frac{\overline{D} - \mu_D}{s_D / \sqrt{n}} = \frac{1.25 - 0}{3.615 / \sqrt{8}} \approx 0.978$$

Decision: Do not reject H_0 .

Conclusion: There very little evidence (p-value > 0.10, p-value ≈ 0.361) to support the claim that there is a significant difference in the pulse rates between identical twins.

The 99% confidence interval is -3.23 < μ_D < 5.73. Notice that this interval includes 0.

10.
$$\hat{p}_1 = \frac{80}{150} = \frac{8}{15} = 0.5\overline{3}$$
 $\hat{p}_2 = \frac{30}{100} = 0.3$ $\overline{p} = \frac{80+30}{150+100} = \frac{11}{25} = 0.44$ $\overline{q} = 0.56$

 $\begin{array}{l} \mathbf{H}_0: p_1 = p_2 \\ \mathbf{H}_1: p_1 \neq p_2 \end{array} \quad \text{Claim: } \mathbf{H}_1 \end{array}$

Test statistic:

Critical value(s):: ± 1.96

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{p} \cdot \overline{q} \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{\left(\frac{8}{15} - 0.3\right) - 0}{\sqrt{0.44 \cdot 0.56 \cdot \left(\frac{1}{150} + \frac{1}{100}\right)}} \approx 3.64$$

Decision: Reject H₀.

Conclusion: There is very strong evidence (p-value < 0.0001, p-value ≈ 0.00027) to support the claim that there is a significant difference between the proportion of people who suffer from lung disease in the two communities.

Chapter 10

- 11. 0.5238; 42.5146
- 12. (a) There is no relationship between the two variables.
 - (b) There is a perfect positive relationship between the two variables; a linear function with positive slope characterizes the relationship. All of the points would be on the line.
 - (c) There is a perfect negative relationship between the two variables; a linear function with negative slope characterizes the relationship. All of the points would be on the line.
- 13. ± 0.514 ; Reject H₀: $\rho = 0$.
- 14. (a) -0.9776 (b) y' = 57.6425 0.0093x
 - (c) 24.7 mpg (d) 21.7 mpg
- 15. (a) $H_0: \rho = 0$ $H_1: \rho \neq 0$ (Claim)

Test Statistic: $r \approx 0.9808$

Critical Value: ±0.444

Decision: Reject H₀

Conclusion: There is an extremely strong positive correlation (p-value $\approx 1.14 \times 10^{-15}$) between ad costs and years.

(b) y' = -276,549,517.2 + 139,144.5511x

(c)

Year	Cost
2015	\$3,800,000
2016	\$4,000,000
2020	\$4,500,000
2025	\$5,200,000

(d) 96.19

(e) 139,144; an average increase of \$139,144 per year for the 30-second ad

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