Statistics Probability

A probability experiment is a process that leads to well-defined results called outcomes.

An event E consists of one or more outcomes of an experiment. $0 \le P(E) \le 1$

3 types of probability:

(1) Classical – Assumes outcomes are equally likely, theoretical
(2) Empirical – Uses actual relative frequencies, "hands on"
(3) Subjective – Educated guess, opinions, complex decisions (many variables)

The complement of an event E is the set of possible outcomes not included in the event. $P\left(\overline{E}\right) = 1 - P(E)$

Given a frequency distribution, the probability of an event being in a particular class is $P(E) = \frac{f}{n}$.

Law of Large Numbers: Empirical probability approaches classical probability as the number of trials increases.

Two events are mutually exclusive if they cannot occur at the same time (i.e., they have no outcomes in common).

Two events are independent if the fact that A occurs does not affect or influence the probability of B occurring.

Exercises:

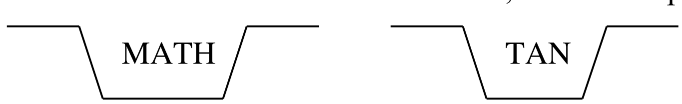
1. Using the given probability of an event, find the probability of its complement.

   (a) $\frac{1}{6}$ _____ (b) 50% _____ (c) 0.72 _____ (d) 0.3 _____

2. A container has six white balls, four red balls and ten blue balls. If one ball is drawn at random, find the probability that

   (a) a red or a white ball is drawn (b) neither a red nor a blue marble is drawn

3. Consider the two containers below. If a container above is selected at random, and then a letter is selected at random from the chosen container, what is the probability that the letter chosen is a vowel?



4. You are sent to prison (though quite innocent). After a few years, the king decides to play a "life or death" game with you. He gives you 5 red marbles, 5 green marbles, and 2 cloth bags. You are allowed to put marbles in the bags as you wish. The king will choose a bag, and then choose a marble from that bag. If the marble he chooses is red, you are executed; if the marble is green, you are set free. If you choose to place 1 green marble in one bag and the rest of the marbles in the other bag, find the probability of your freedom. Round to the nearest percent.

<u>Tree diagrams</u>: In a sequence of n events in which the first has k1 possibilities, the second has k2, the third has k3, and so on, the total possibilities of the sequence will be $k_1 \cdot k_2 \cdot k_3 \cdots k_n$.

<u>Permutations</u>: an arrangement of n objects in a specific order

The number of permutations of n distinct objects taken all together is n!

The number of permutations of n objects, where k_1 are alike, k_2 are alike, etc. is $\frac{n!}{k_1! \cdot k_2! \cdot k_3! \cdots k_p!}$, where $k_1 + k_2 + k_3 + \cdots + k_p = n$

The number of permutations of n distinct objects taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$.

<u>Combinations</u>: the selections of distinct objects without regard to order

The number of combinations of n distinct objects taken r at a time is ${}_nC_r = \frac{n!}{r!(n-r)!}$

<u>Exercises</u>:

5. If blood types can be A, B, AB, or O, and Rh+ or Rh–, draw a tree diagram for the possibilities.

6. A combination lock consists of the numbers 0 to 59. If no number can be used twice, how many different combinations are possible using three numbers? Remember, a combination lock is really a permutation lock.

7. How many different standard license plates can be made currently in the state of Georgia? (3 letters followed by 4 digits, with repetition allowed) What if repetition for letters and digits is not allowed?

   (a) (b)

8. In how many ways can a committee of 4 be selected from a club with 12 members?

9. Given a class of 12 girls and 8 boys, what is the probability that a committee of 5, chosen at random, consists of 3 girls and 2 boys? Give an exact answer, and also round to the nearest percent.

Statistics **Probability Distributions**

A <u>random variable</u> is a variable whose values are determined by "chance".

E.g., # of phone calls, # of joggers (discrete); weight, time, temperature (continuous)

A <u>probability distribution</u> consists of the values of a random variable and the probabilities which correspond. These probabilities are determined theoretically or by observation.

x, p(x) Two requirements: (1) $\Sigma\, p(x) = 1$ (2) $0 \le p(x) \le 1$

The <u>mean</u> or <u>expected value</u> of a random variable of a probability distribution is $\mu = \Sigma\left(x \cdot p(x)\right)$, where $x_1, x_2, x_3, \ldots, x_n$ are the outcomes and $p(x_1), p(x_2), p(x_3), \ldots, p(x_n)$ are the corresponding probabilities. If $\mu = 0$, a game is said to be fair.

The <u>standard deviation</u> is given by $\sigma = \sqrt{\Sigma\left[(x-\mu)^2 \cdot p(x)\right]}$ or $\sqrt{\Sigma\left[x^2 \cdot p(x)\right] - \mu^2}$.

A <u>binomial distribution</u> is a probability experiment with these characteristics:

1. Each trial has 2 outcomes ("success" or "failure").
2. There must be a fixed number of trials.
3. The outcome of each trial must be independent of one another.
4. The probability of a success must remain the same for each trial.

   P("success") = p P("failure") = q

In a binomial experiment, the probability of exactly x successes in n trials is given by $P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$.

For a binomial distribution, the mean is $\mu = n \cdot p$ and the standard deviation is $\sigma = \sqrt{n \cdot p \cdot q}$.

<u>Exercises</u>:

10. Given the following distribution:

X	1	2	3	4	5	6
P(X)	0.3	0.1	0.1	0.1	0.2	0.2

(a) Determine whether it is a probability distribution.
(b) Construct a graph for the distribution.
(c) Find the mean and standard deviation for the distribution. Round to the nearest tenth.

11. Which of the following are binomial experiments?

(a) _____ Surveying 100 people to see if they prefer Sudsy Soap over other brands.
(b) _____ Surveying 100 families to determine the number of cell phones each household owns.
(c) _____ Asking 50 people whether or not they smoke.
(d) _____ Tossing a coin 100 times to see how many heads occur.
(e) _____ Testing 4 different brands of aspirin to see which brands are effective.

12. If 80% of the applicants are able to pass a driver's proficiency road test, find the mean and standard deviation of the # of people who pass the test in a sample of 300 applicants.