The Birthday Problem*

How many people are needed in a group in order for the probability to favor 2 of them having the same birthday (month and day)?

Critical ideas:

- The event that something happens and the event that something does not happen are called complementary events. Probabilities of complementary events always add to 1.
- When the outcome of one event doesn't influence another event's likelihood of occurring, the two events are said to be independent. For two independent events, A and B, the probability of A occurring and then B occurring is the product of the individual probabilities.

Since either there are 2 people in a group who share the same birthday or no 2 people do, these are complementary events and

P(at least 2 have the same birthday) = 1 – P(no 2 have the same birthday)

We'll focus on the right-hand side, assuming that no 2 have the same birthday. As we then gather information from a group, the birthday of the "first person" could be any of the 365 days in the year; the "second" could be any of the remaining 364 days; the "third" could be any of the 363 remaining day; and so on.

Because the probability of several independent events happening in succession is found by multiplying the probabilities of each of them,

$$\text{P(no 2 have the same birthday)} = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \ldots$$

(one fraction for each person in the group)

and $$\text{P(at least 2 have the same birthday)} = 1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \ldots$$

The probabilities that there are 2 people in a group who share the same birthday and that no 2 people share the same birthday are shown in the table below. Each probability is rounded to the nearest percent. Check out the results.

Number of people in group	Probability that all birthdays are different (no 2 share the same birthday)	Probability that 2 people share the same birthday
2	100%	0%
4	98%	2%
6	96%	4%
8	93%	7%
10	88%	12%
12	83%	17%
14	78%	22%

16	72%	28%
18	65%	35%
20	59%	41%
22	52%	48%
23	50%	50%
24	46%	54%
26	40%	60%
28	35%	65%
30	29%	71%
32	25%	75%
34	20%	80%
36	17%	83%
38	14%	86%
40	11%	89%
42	9%	91%
44	7%	93%
46	5%	95%
48	4%	96%
50	3%	97%
72	1%	99%
366 or more	0%	100%

A group of just 24 people gives a probability of over 50% that at least 2 people will share the same birthday; a group of 32 people gives a probability of around 75% that at least 2 people will share the same birthday; a group of 72 people gives a probability of around 99% that at least 2 people will share the same birthday. Are you surprised?

The birthdays of the first 30 Presidents of the United States are included below. Notice in this group of 30 that Presidents Polk and Harding share the same November 2 birthday.

President	Birthday	President	Birthday
1. Washington	February 22	16. Lincoln	February 12
2. J. Adams	October 30	17. A. Johnson	December 29
3. Jefferson	April 13	18. Grant	April 27
4. Madison	March 16	19. Hayes	October 4
5. Monroe	April 28	20. Garfield	November 19
6. J.Q. Adams	July 11	21. Arthur	October 5
7. Jackson	March 15	22. Cleveland	March 18
8. Van Buren	December 5	23. B. Harrison	August 20
9. W.H. Harrison	February 9	24. McKinley	January 29
10. Tyler	March 29	25. T. Roosevelt	October 27
11. Polk	November 2	26. Taft	September 15
12. Taylor	November 24	27. Wilson	December 28
13. Fillmore	January 7	28. Harding	November 2
14. Pierce	November 23	29. Coolidge	July 4
15. Buchanan	April 23	30. Hoover	August 10

*Adapted with permission from *Mathematics: A Human Endeavor* by Harold R. Jacobs. W.H. Freeman and Company, 1982.