Is the exponential function one-to-one? \_\_\_\_\_\_\_ So, does it have an inverse function? \_\_\_\_\_\_\_

Then find and graph its inverse generally. Then use the specific example below.

(Assume a > 1)

1. (a) Complete the tables below for the given exponential and logarithmic functions.

y = 2x y = log2x

(b) Graph these two functions along

with y = x on the coordinate

grid. Include any asymptote(s)

and intercept(s).

(c) What do you notice about the

tables in part (a)?

(d) What do you notice about the graphs?

(e) Notice also that the domain of the log function = the range of the exponential function = \_\_\_\_\_\_\_\_\_\_\_

And the range of the log function = the domain of the exponential function = \_\_\_\_\_\_\_\_\_\_\_.

Conversions from one form to another

Use **logax = y ↔ ay = x**  
  
2. Complete the chart, converting the given equation from one form to the other.

|  |  |
| --- | --- |
| **Logarithmic form** | **Exponential form** |
| (a) |  |
| (b) log 0.001 = -3 |  |
| (c) |  |
| (d) |  |
| (e) |  |
| (f) |  |
| (g) ln 39 ≈ 3.6636 |  |

3. Evaluate without a calculator. Give exact answers, whenever possible.

(a)  (b) log 100,000,000 (c)  (d) log (–10)

(e) (f)  (g) log 0.000001

4. For the logarithmic function given by , give a rough sketch. Include any asymptotes and intercepts.

How does this graph compare with  in shape and location? Refer to the graph of  that we developed earlier in this section.

**Calculator Keys:**

Common logarithm LOG = log10 Natural logarithm LN = loge

5. Evaluate. Use your calculator to approximate these to 4 decimal places.

(a) log 153 (b) log 10,000 (c) ln 44 (d) ln 250,000

**Solving logarithmic equations**:

Use (1) changing forms or (2) **logau = logav ↔ u = v**

6. Solve for x. Give exact answers, if possible.

(a)  (b)  (c) 

(e)  (e) 

**Applications**:

1. In chemistry, the acid potential of aqueous solutions is measured in terms of the pH scale. Tremendous swings in hydrogen ion concentration occur in water when acids or bases are mixed with water. These changes can be as big as . Since the pH is a logarithmic scale, every multiple of ten in  concentration equals one unit on the scale. Physically, the pH is intended to tell what the acid potential is for the solution. The pH values range from negative values to numbers above 14. The pH of a substance is defined as , where  is the hydrogen ion concentration, measured in moles per liter. Pure water is neutral and has a pH of 7; acids have a pH lower than 7; bases have a pH higher than 7. Use the formula to complete the chart. Round like the given values.

|  |  |  |
| --- | --- | --- |
| **Substance** | **pH value** | **in mol/L** |
| (a) Eggs |  |  |
| (b) Apple | 3.2 |  |
| (c) Household ammonia | 11.6 |  |
| (d) Milk |  |  |

2. The magnitude R, measured on the Richter scale, of an earthquake of intensity I is defined as , where  is the threshold intensity for the weakest earthquake that can be recorded on a seismograph. Complete the following table. In each case, round your answers like the given values.

|  |  |  |
| --- | --- | --- |
| **Earthquake** | **Intensity, I** | **Richter Number, R** |
| (a) Chile, 1960 |  | 9.6 |
| (b) Italy, 1980 |  |  |
| (c) San Francisco, 1989  (during a MLB World Series) |  |  |
| (d) Kobe, Japan, 1995 |  | 6.8 |
| (e) Indian Ocean, 2005 |  | 9.0 |
| (f) Haiti, 2010 |  | 7.0 |
| (g) Chile, 2010 |  | 8.8 |

3. The loudness L, in decibels (after Alexander Graham Bell), of a sound of intensity I is defined to be , where  is the minimum intensity detectable by the human ear (such as the tick of a watch at 20 feet under quiet conditions).  is measured to be 20 micropascals (μPa), or 0.02 mPa or . (This is a very low pressure; it is 2 ten-billionths of an atmosphere. Nevertheless, this is about the limit of sensitivity of the human ear, in its most sensitive range of frequency. Usually this sensitivity is only found in rather young people or in people who have not been exposed to loud music or other loud noises.) If a sound is 10 times as intense as another, its loudness is 10 decibels greater; if a sound is 100 times as intense as another, its loudness is 20 decibels greater; and so on.

Complete the following table. Round like the given values.

|  |  |  |
| --- | --- | --- |
| **Sound** | **Intensity, I** | **Decibel Number** |
| (a) A whisper |  | 18 |
| (b) Pain threshold for a human ear |  |  |
| (c) A crowded restaurant |  | 80 |
| (d) A library |  |  |