An Interesting Set of Problems

The formula for the accumulated amount of an investment (or loan) is given by the formula \( A = P \left(1 + \frac{r}{n}\right)^{nt} \), where \( P \) is the principal, \( r \) represents the annual interest rate, \( n \) is the number of interest periods in a year, and \( t \) is the number of years. In all your calculations, it is customary to round accumulated amounts (\( A \)) down to the nearest penny. When plotting the points and corresponding curve, place \( A \), the dependent variable, on the vertical axis, and place the independent variable on the horizontal axis. Then analyze the table, graph and formula to determine features of the function. Choose any two of the 4 cases below. In part (a), show your work on one of the computations.

1. Find the accumulated amount on an investment of \( P \) at 7% annual interest, compounded quarterly over 8 years.
   
   (a) Let \( P \) be \$1,000, \$2,000, \$3,000, \$4,000, and \$5,000. Make and complete a table showing these \( P \) values with the corresponding \( A \) values. [20 points]
   
   (b) Sketch the graph of \( P \) vs. \( A \). \( A \) is the dependent variable. [10 points]
   
   (c) What type of function is it? (linear, polynomial, rational, exponential, logarithmic, radical, or a combination of these) [4 points]
   
   (d) Describe features of the functions using the following terms: intercept, slope, vertex, asymptote, maximum, minimum, symmetry (whichever applies). [8 points]
   
   (e) As \( P \) increases, \( A \) ______________________________. (Fill in the blank with: increases steadily, increases by more and more, increases by less and less, decreases steadily, decreases by more and more, or decreases by less and less.) [4 points]
   
   (f) Write the specific equation relating \( P \) and \( A \). (\( A \) as a function of \( P \)) [4 points]

2. Find the accumulated amount on an investment of \$2,500 at \( r\% \) annual interest, compounded quarterly over 5 years.
   
   (a) Let \( r \) be 2, 4, 6, 8, and 10. Make and complete a table showing these \( r \) values with the corresponding \( A \) values. [20 points]
   
   (b) Sketch the graph of \( r \) vs. \( A \). \( A \) is the dependent variable. [10 points]
   
   (c) What type of function is it? (linear, polynomial, rational, exponential, logarithmic, radical, or a combination of these) [4 points]
   
   (d) Describe features of the functions using the following terms: intercept, slope, vertex, asymptote, maximum, minimum, symmetry (whichever applies). [8 points]
   
   (e) As \( r \) increases, \( A \) ______________________________. (Fill in the blank with: increases steadily, increases by more and more, increases by less and less, decreases steadily, decreases by more and more, or decreases by less and less.) [4 points]
(f) Write the specific equation relating r and A. (A as a function of r)  [4 points]

3. Find the accumulated amount on an investment of $2,500 at 5% annual interest, compounded n times each year over 5 years.

(a) Let m be 1, 4, 12, 52, and 360. Make and complete a table showing these n values with the corresponding A values.  [20 points]

(b) Sketch the graph of n vs. A. A is the dependent variable.  [10 points]

(c) What type of function is it? (linear, polynomial, rational, exponential, logarithmic, radical, or a combination of these)  [4 points]

(d) Describe features of the functions using the following terms: intercept, slope, vertex, asymptote, maximum, minimum, symmetry (whichever applies).  [8 points]

(e) As n increases, A ____________________________ (Fill in the blank with: increases steadily, increases by more and more, increases by less and less, decreases steadily, decreases by more and more, or decreases by less and less.)  [4 points]

(f) Write the specific equation relating n and A. (A as a function of n)  [4 points]

For 4 BONUS points (An extension of case 3) — What does the value of A approach as n approaches infinity (i.e., as the bank compounds "continuously")?

4. Find the accumulated amount on an investment of $1,000 at 4.5% annual interest, compounded monthly over t years.

(a) Let t be 5, 10, 15, 20, and 40. Make and complete a table showing these t values with the corresponding A values.  [20 points]

(b) Sketch the graph of t vs. A. A is the dependent variable.  [10 points]

(c) What type of function is it? (linear, polynomial, rational, exponential, logarithmic, radical, or a combination of these)  [4 points]

(d) Describe features of the functions using the following terms: intercept, slope, vertex, asymptote, maximum, minimum, symmetry (whichever applies).  [8 points]

(e) As t increases, A ____________________________ . (Fill in the blank with: increases steadily, increases by more and more, increases by less and less, decreases steadily, decreases by more and more, or decreases by less and less.)  [4 points]

(f) Write the specific equation relating t and A. (A as a function of t)  [4 points]

Worth up to 100 points  Due Date: _______________________

Do Your Best! Live and Learn!