Annuities

Here's a variation on the compound interest/future value problem we have discussed (with some resemblance to the monthly payment scenario).

A series of equal payments is called an **annuity**, and an **ordinary annuity** is an annuity with periodic payments made at the end of each period. The compound interest problems we've handled have assumed a one-time initial investment and no deposits (or withdrawals) during the full course of the investment. The annuity is a method to systematically save or invest, and many financial counselors recommend this technique, both for the discipline and for the fantastic results. This technique in the stocks and bond realm involves "dollar cost averaging". You may want to do a little research to find out more about how annuities work.

Example #1: A family enters a savings plan to invest \$1000 at the end of each year for 5 years. The annuity will pay 7% interest, compounded annually. Find the value of the annuity at the end of the 5 years.

Solution:

Since \$1000 is deposited each year, the first deposit will draw interest longer than subsequent deposits. The value of each deposit at the end of 5 years is the original \$1000 plus the compound interest for the time it draws interest. The chart below summarizes how payments of \$1000 deposited at the end of each year grow over a 5-year period.

Year deposited	**Length of time deposit draws interest**	**Value of deposit at the end of 5 years**
1	4 years	$1000\ (1.07)^4 \approx \$1,310.80$
2	3 years	$1000\ (1.07)^3 \approx \$1,225.04$
3	2 years	$1000\ (1.07)^2 \approx \$1,144.90$
4	1 year	$1000\ (1.07) \approx \$1,070.00$
5	0 years	$1000\ (1.07)^0 \approx \$1000.00$

The final value is obtained by adding the 5 payments and interest accumulated on each one over the 5 years (\$1,310.80 + \$1,225.04 + \$1,144.90 + \$1,070 + \$1000 = \$5,750.74).

Clearly, this method for calculation is tedious, so we have a more straightforward formula.

$$\boxed{A = P \cdot \frac{(1+i)^n - 1}{i}}$$

where A = the future value of the investment(s)

P = the periodic payment
i = the periodic interest rate
n = the total number of periods

On the example above, we could calculate the accumulated amount:

$$A = 1000 \cdot \frac{(1+.07)^5 - 1}{.07} \approx \$5,750.74.$$

Example #2: A family enters a savings plan to invest \$25 at the end of each month for 10 years. The annuity will pay 3.5% interest, compounded monthly. Find the value of the annuity at the end of the 10 years.

Solution: P = \$25, i = 0.035/12, n = 10 x 12

$$A = 25 \cdot \frac{(1 + .035/12)^{120} - 1}{.035/12} \approx \$3{,}585.81$$

<u>Exercises</u>: In each problem, round down to the nearest hundredth.

1. A family pays \$800 at the end of every 6 months for 7 years into an ordinary annuity paying 10% annual interest, compounded semiannually. Find the future value at the end of 7 years.

2. A young man deposits \$500 into an account every 3 months for 2 years. The money earns 6% compounded quarterly. Determine the value of each deposit at the end of 2 years and the total amount in the account.

3. A young lady saves up \$2,000 and invests it at the end of each year in an IRA. The investment earns 8% annual interest. If she continues doing this every year for 40 years, what is the future value of her investments.

4. You invest \$100 each month (automatically from your paycheck) in a stock mutual fund and earn a 12% annual rate of return.

   (a) After 10 years, how much have you deposited?

   (b) What is the future value of this investment plan?

<u>Solutions</u>:

1. \$15,678.90 2. \$4,216.41 3. \$518,113.03

4. (a) \$12,000 (b) \$23,003.86