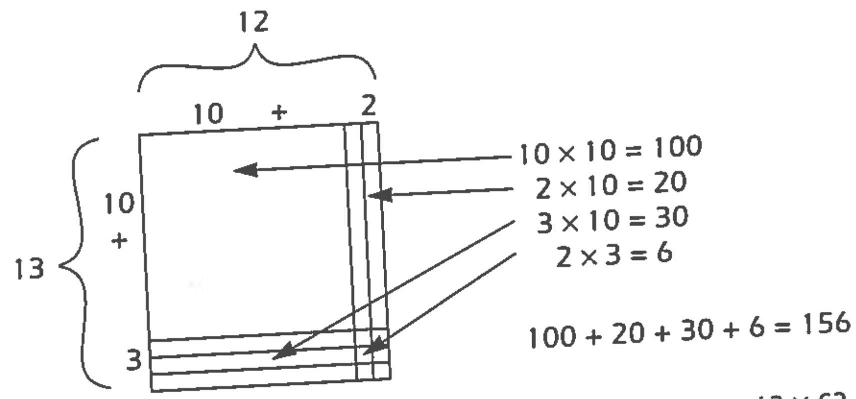
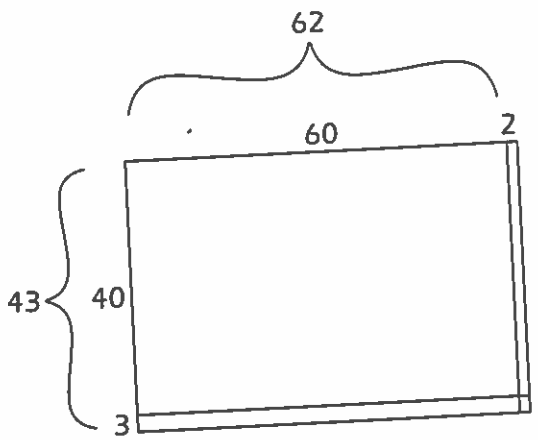
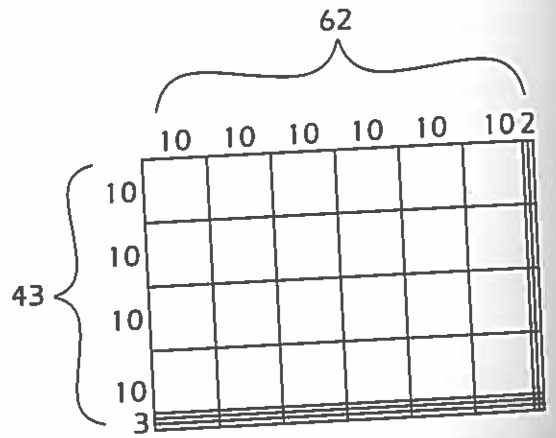
These areas are then added together. (Base ten blocks—hundreds, tens, and ones—can be used to fill in the regions to represent the areas concretely.)



Try the rectangular array approach with Sasha's second problem, 43 × 62. One side of the 43 by 62 rectangle can be marked to show 40 + 3, or 10 + 10 + 10 + 10 + 3. The other side can be marked to show 60 + 2, or 10 + 10 + 10 + 10 + 10 + 10 + 2.





Emily broke down 12 × 13 into computations simple enough to do in her head. She used a method similar to the partial product algorithm in that she found some partial products and then added. Her method also relies on the distributive property, distributing 13 across the 10 and the 2: (13 × 10) + (13 × 2).

12 × 13 ⇒ 10 × 13 = 130 ⇒ 2 × 13 = 26 ⇒ 130 + 26 = 156

Emily's solution to 43 × 62 again uses the distributive property, but in order to keep the calculations easy enough to do in her head she first thought of 43 as (20 + 20 + 3) and then calculated the partial products of each of these multiplied by 62. Since there were many steps in this calculation, she jotted them down:

20 × 62 = 1240
20 × 62 = 1240
3 × 62 = 186
1240 + 1240 + 186 = 2666

Notice that the individual procedures produce individual partial products, which are then added to find the final product. Clearly, Emily understands what happens when one multiplies and is ready to learn a quicker, more efficient algorithm.

Tabitha used the *lattice* method of multiplication. It is an alternative algorithm students almost never invent on their own; it is taught because analyzing it can help students make sense of the role of regrouping in multidigit multiplication. To multiply 43 × 62, Tabitha wrote the two factors, 43 and 62, above and to the right of the lattice and recorded partial products in cells with the tens value above the diagonal line and the ones value below the diagonal line (see 3 × 6 and 3 × 2 in the figure on page 49). After she had done this for 4 × 6 and 4 × 2 as well, she extended the diagonal lines and added the numbers in each diagonal. If the sum in a diagonal was greater than 9 (such as the second diagonal from the right), she regrouped the 10 tens as 1 hundred into the next diagonal to the left. The final product is read starting from the left side of the lattice. The product of 43 × 62 is 2666.

Why does the lattice method of multiplication work? While it looks very different, this algorithm is similar to the standard multiplication algorithm. The number 43 is multiplied by 2 (the bottom row) and by 60 (the top row) for a total of 62 times. While it appears that you are only multiplying by 6, notice how the numbers in the cells in the top row are all shifted over one place because of the diagonals. This has the effect of placing the numbers in positions that represent multiplication by 60, not 6. In the standard multiplication algorithm we also shift the placement of digits (often using a zero as a place holder) in order to represent multiplying by 60:

```
   43
 × 62
   86
 2580 ⇐ shift over or place a 0 in the ones place
 2666
```

One feature of the lattice algorithm that makes it especially appealing to some students is that multiplication and addition within the algorithm are kept separate. Basic facts are used to fill the cells but adding only occurs when determining the sums of the diagonals. ▲

Activity

Analyzing Students' Thinking, Division

Objective: learn some common division strategies.

Examine the following examples of students' procedures for solving division problems. What did each student do to obtain a correct answer? Why does the student's algorithm work?

Doug

```
  137 r 4
5)689
 −500 | 100 • 5
  189
   50 |  10 • 5
  139
   50 |  10 • 5
   89
   50 |  10 • 5
   39
   35 |   7 • 5
    4   137 • 5
```

Nancy

```
  137 r 4
5)689
  5
  18
  15
   39
   35
    4
```

Madelaine

```
500 ÷ 5 = 100
100 ÷ 5 =  20
 50 ÷ 5 =  10
 30 ÷ 5 =   6
  9 ÷ 5 =   1 r 4
689 ÷ 5 = 137 r 4
```