

Table 1. Common addition and subtraction situations.

Join
Separate
P=Whole

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
Put Together/ Take Apart ²	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
Compare ³	("How many more?" version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? $2 + ? = 5$, $5 - 2 = ?$	("Version with 'more'") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $5 + 2 = ?$	("Version with 'more'") Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$
	("How many fewer?" version) Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	("Version with 'fewer'") Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $5 + 2 = ?$	("Version with 'fewer'") Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

P=U (P unknown)

Quantity

Referent

These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "takes" or "results in" but always does mean "is the same number as."

Filter: Addend can be unknown, so there are three variations of each problem situation. Both Addends Unknown is a productive extension of this situation, especially for small numbers less than or equal to 10.

For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

²Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations¹

	M ↓ Unknown Product $3 \times 6 = ?$	P.P. ↓ Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	M.O. ↓ Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, ² Area ³	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

¹The language in the array examples shows the easiest form of a "by" problem. A harder form is to use the terms rows and columns. The apples in the nearby window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

²Area involves arrays of squares that have been counted together, so that there are no gaps or overlaps, so any rectangles include these especially important measurement situations.

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³The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.