CGI: Children's Solution Strategies

As the most basic strategies, children use physical objects (counters), pictures, tally marks, or fingers to directly model the action or relationships given in the problem.

Over time, children's strategies become more abstract and efficient. They replace direct modeling strategies with counting strategies which in turn are replaced by number fact strategies or invented algorithms.

I. Direct Modeling

Problem	Strategy Description
Join (Result Unknown) Ellen had 3 tomatoes. She picked 9 more tomatoes in her momma's garden. How many tomatoes does she have now?	Joining All The child constructs a set of 3 objects and a set of 9 objects. Then they find the answer by counting all the objects in the two sets.
Join (Change Unknown) Chuck has 9 dollars. How many more dollars does he need to buy a stuffed animal that costs 12 dollars?	Joining To The child constructs a set of 9 objects. Then they add objects to this set until there is a total of 12 objects. They find the answer by counting the number of objects added.
Separate (Result Unknown) There were 12 otters playing in the water. Nine otters swam away. How many otters were still playing in the water?	Separating From The child constructs a set of 12 objects and then removes 9 objects. They find the answer by counting the remaining objects.
Separate (Change Unknown) There were 12 children on the school bus. Some children got off. Now there are 8 children on the bus. How many children got off the bus?	Separating To The child counts out a set of 12 objects. Then they remove objects from the set until the number of objects remaining is equal to 8. Then they find the answer by counting the objects they removed.
Compare (Difference Unknown) Megan has 4 stickers. Randy has 11 stickers. How many more stickers does Randy have than Megan?	Matching The child makes a set of 4 objects and a set of 11 objects. The two sets are matched one-to-one until one set is used up. They find the answer by counting the unmatched objects remaining in the larger set.
Join (Start Unknown) Alyssa had some books. She went to the library and got 6 more books. Now she has 15 books altogether. How many books did she have to start with?	Trial and Error The child constructs a set of objects, adds a set of 6 objects to the set, and counts the objects in the resulting set. If the final count is 15, then the number of objects in the initial set is the answer. If it is not 15, they try a different initial set.
Multiplication Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does he have in all?	Grouping Make 4 groups with 6 counters in each group. Count all the counters to find the answer.
Measurement Division Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	Measurement Put 24 counters into groups with 6 counters in each group. Count the groups to find the answer.
Partitive Division Bart has 6 boxes of pencils with the same number of pencils in each box. Altogether he has 24 pencils. How many pencils are in each box?	Partitioning/Dealing Divide 24 counters into 6 groups with the same number of counters in each group. Count the counters in one group to find the answer.

II. Counting

Problem	Strategy Description
Join (Result Unknown) Ellen had 3 tomatoes. She picked 9 more tomatoes in her momma's garden. How many tomatoes does she have now?	Counting On From First The child begins counting with 3 and continues on for 9 more counts. The answer is the last number in the counting sequence (i.e., 4, 5, 6, 7, 8, 9, 10, 11, 12; 12 is the answer).

Join (Result Unknown) Ellen had 3 tomatoes. She picked 9 more tomatoes in her momma's garden. How many tomatoes does she have now?	Counting On From Larger The child begins counting with 9 and continues on for 3 more counts. The answer is the last number in the counting sequence (i.e., 10, 11, 12; 12 is the answer).
Join (Change Unknown) Chuck has 9 dollars. How many more dollars does he need to buy a stuffed animal that costs 12 dollars?	Counting On To The child counts forward starting from 9 and continues until reaching 12. The answer is the number of counting works in the sequence (i.e., 10, 11, 12; the answer is 3).
Separate (Result Unknown) There were 12 otters playing in the water. Nine seals swam away. How many otters were still playing in the water?	Counting Down The child counts backward starting from 12. The sequence continues for 9 more counts. The last number in the counting sequence is the answer (i.e., 11, 10, 9, 8, 7, 6, 5, 4, 3; 3 is the answer).
Separate (Change Unknown) There were 12 children on the school bus. Some children got off. Now there are 8 children on the bus. How many children got off the bus?	Counting Down To The child counts backward from 12, and continues until reaching 8. The answer is the number of counting words in the sequence (i.e., 11, 10, 9, 8; the answer is 4).
Multiplication Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does he have in all?	Repeated Addition/Skip Counting The child counts 4, 8, 12, 16, 20, 24. With each count, the child extends one finger. When they have extended 6 fingers, they stop. The answer is 24.
Measurement Division Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	Repeated Subtraction/Skip Counting The child counts 4, 8, 12, 16, 20, 24. With each count, the child extends one finger. When they have counted to 24, they notice that they have extended 6 fingers. The answer is 6.
Partitive Division Bart has 6 boxes of pencils with the same number of pencils in each box. Altogether he has 24 pencils. How many pencils are in each box?	Trial and Error Partitioning/Dealing The child counts 3, 6, 9, 12, 15, 18, with a finger extended for each count until they reach 6 and decides that 3 is not enough. Then they count 4, 8, 12, 16, 20, 24. The answer is 4.

III. Number Facts/Invented Algorithms

Students display base-10 understanding as they invent, derive, and recall number facts. Encourage and build on this creativity and intuition! The list below is a small sampling of what students may invent.

Decade Counting/ Incrementing 58 + 47 50, 60, 70, 80, 90, 98, 100, 105
+10 +10 +10 +10 +8 +2 +5

Compensating 58 + 47 = (58 + 2) + (47 – 2) = 60 + 45 = 105

83 – 38 = (83 + 2) – (38 + 2) = 85 – 40 = 45

Combining 10s and 1s 58 + 47 50 + 40 = 90 8 + 7 = 15 90 + 15 = 105

Incrementing 58 + 47 58, 68, 78, 88, 98, 100, 105
+10 +10 +10 +10 +2 +5

83 – 38 83 – 30 = 53 53 – 3 = 50 50 – 5 = 45

Distributive Property $9 \times 32 = 9 \times (30 + 2) = 9 \times 30 + 9 \times 2 = 270 + 18 = 288$

Partial Products $58 \times 47 = 50 \times 40 + 50 \times 7 + 8 \times 40 + 8 \times 7$
$= 2000 + 350 + 320 + 56 = 2{,}726$

Source: Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's mathematics: Cognitively guided instruction.* Heinemann, Portsmouth, NH.