**Cognitively Guided Instruction  
Principles of Development of Base-Ten/Place-Value Understanding**  
(Carpenter et al., 1999, Chapter 6)

Children must develop an understanding that collections of ten can be counted.

The primary contexts for developing this understanding are found in multiplication and division situations involving groups of tens.

If children can count, they can model and solve word problems involving addition, subtraction, multiplication, and division with two- and three-digit numbers. These experiences provide the foundation needed to understand the positional notation required by the formal computational algorithms.

**Evidence of Base-Ten Understanding**

**Counting a group of ten as a unit**:

**Decade counting/counting by tens**: “10, 20, 30” (pointing to groups of ten)  
 **Counting tens as ones**: “1, 2, 3, 30” (pointing to groups of ten)  
 **Incrementing by tens**: “28, 38, 48, 49” (for 18 + 31 = \_\_\_ )

**Showing a unit of ten as equal to ten individual units**:

Breaking apart or building a rod of 10 Unifix cubes  
 Exchanging a Base-10 rod for 10 units or vice versa

**Replacing tens with combinations of numbers that sum to ten**:

6 + 4, 7 + 3, 8 + 2, etc.

**Recognizing groupings in spoken number words**:

“Thirty-two is three tens and two ones.”  
 “Three hundred four is three hundreds and four ones.”

**Direct place-value explanation (DPVE)**:

“Forty plus five is forty-five.”

**Recognizing the positional values of the digits 0-9 in written numerals**:

“The digits in the number 304 represent three hundreds, zero tens, and four ones.”

**Using invented algorithms/mental strategies that depend on base-ten understanding**:

For the problem 28 + 35 =\_\_\_ ,

**Incrementing**:“20 and 30 is 50; 8 more is 58; 2 more is 60; and 3 more than that is 63.”

**Combining Tens and Ones**: “20 and 30 is 50; 8 plus 5 is 13; and 50 plus 13 is 63.”

**Compensating**: “30 and 35 would be 65; but it’s 28, not 30, which is 2 less; so it’s 63.”

**Cognitively Guided Instruction: Base-10 Understanding**

1. There are three bags of baseballs with 10 baseballs in each bag. There are also 6 more baseballs. How many baseballs are there altogether?

2. Mrs. Allen has 4 packages of cookies. There are 10 cookies in each package. She also has 4 other cookies. How many cookies does she have in all?

3. Jamaka has 7 bags of beads. There are 10 beads in each bag. She also has 5 extra beads. How many beads does she have in all?

4. Matt has 32 pennies. He puts 10 pennies into each box. How many boxes can he fill with these 32 pennies?

5. Jamaal has 53 stamps in his stamp collection. He puts them in a book with 10 stamps on each page. How many pages does he need if all the stamps are put in the book?

6. The second graders had 64 balloons for the school party. They tied these balloons into bunches of 10. How many bunches could they make? How many were left over?

7. Max had 37 comic books. For his recent birthday, his parents gave him 25 more comic books. How many comic books does he have now?

8. Tiffany’s mom ordered 50 cupcakes for her party. At the party, her friends ate 32 of the cupcakes. How many does she have left?

9. Willy has 42 crayons. Melissa has 26 crayons. How many more crayons does Willy have than Melissa?

10. Earl had a collection of 37 seashells. On his recent trip to the beach, he found lots of seashells by the seashore. Now he has 52 seashells. How many did he find at the beach?

11. Some people were at a recreational league soccer game. There were 49 children and 36 adults. How many people were at the soccer game altogether?

12. Amelia’s family has 53 books. Mark’s family has 29 more books than Amelia’s family. How many books does Mark’s family have?

13. There are 3 bags of soccer balls with 10 soccer balls in each bag. There are also 7 more soccer balls. How many soccer balls are there altogether?

14. Jamaal has 59 stamps in his stamp collection. Marcus has 3 times as many as Jamaal in his stamp collection. How many stamps does Marcus have?

15. Tammy had 38 dollars. One weekend, she earned 25 dollars raking leaves for her neighbors. How much money did Tammy have then?