**Cognitively Guided Instruction  
Principles of Development of Base-Ten/Place-Value Understanding**  
(Carpenter et al., 1999, Chapter 6)

Children must develop an understanding that collections of ten can be counted.

The primary contexts for developing this understanding are found in multiplication and division situations involving groups of tens.

If children can count, they can model and solve word problems involving addition, subtraction, multiplication, and division with two- and three-digit numbers. These experiences provide the foundation needed to understand the positional notation required by the formal computational algorithms.

**Evidence of Base-Ten Understanding**

**Counting a group of ten as a unit**:

**Decade counting/counting by tens**: “10, 20, 30” (pointing to groups of ten)  
 **Counting tens as ones**: “1, 2, 3, 30” (pointing to groups of ten)  
 **Incrementing by tens**: “28, 38, 48, 49” (for 18 + 31 = \_\_\_ )

**Showing a unit of ten as equal to ten individual units**:

Breaking apart or building a rod of 10 Unifix cubes  
 Exchanging a Base-10 rod for 10 units or vice versa

**Replacing tens with combinations of numbers that sum to ten**:

6 + 4, 7 + 3, 8 + 2, etc.

**Recognizing groupings in spoken number words**:

“Thirty-two is three tens and two ones.”  
 “Three hundred four is three hundreds and four ones.”

**Direct place-value explanation (DPVE)**:

“Forty plus five is forty-five.”

**Recognizing the positional values of the digits 0-9 in written numerals**:

“The digits in the number 304 represent three hundreds, zero tens, and four ones.”

**Using invented algorithms/mental strategies that depend on base-ten understanding**:

For the problem 28 + 35 =\_\_\_ ,

**Incrementing**:“20 and 30 is 50; 8 more is 58; 2 more is 60; and 3 more than that is 63.”

**Combining Tens and Ones**: “20 and 30 is 50; 8 plus 5 is 13; and 50 plus 13 is 63.”

**Compensating**: “30 and 35 would be 65; but it’s 28, not 30, which is 2 less; so it’s 63.”