

Recursion Theorem presentation script

Part 1:

A long time ago I was in the library of an ancient castle, where Merlin the magician was showing me his bookshelf. It was a large bookshelf that seemed to stretch on forever, filled with dusty volumes. Most of the books had a blank spine, but on a few a number was printed. I noticed that the bookshelf had small numbers written on it, too, labeling each spot where a book could fit.

“Look at spot 1152”, Merlin suggested

I walked over and saw that the the book there had 1152 printed on its jacket.

“There is magic in that shelf.” he said. “There is always at least one book that knows where it belongs, no matter how you they are arranged.”

“Really?”

“Stand back a little. Alphabetical order” Merlin commanded. The books flew about until the entire shelf was alphabetized. I started looking for a match when he suggested “Try spot 348.” Sure enough it was there.

“How do you know where to look?” I asked.

“There is a mathematical formula that tells you,” he replied. I tried every way of ordering the books I could think of, but the result was always the same. There was always matches and Merlin knew where to find them.

“What is in these books?” I wondered.

“Computer programs.”

I was a bit surprised. “They look older than that.”

“Well they didn’t always have computer programs,” Merlin explained. “Many years ago they were filled with diagrams for Turing machines. Before that they had lengthy verbal description of computations. The magic isn’t in computers, it is in the nature of computation itself.”

I then asked the question anyone in my position would ask, “Can this be used in mathematical research?”

“That’s the idea” Merlin replied, “but first, lets catch the audience up on the background and definitions.”

Part 2:

After a pause, Merlin motioned me back towards the bookcase. “Open one of the books with a number on the spine,” he suggested. I tried book 29 first, but the program in it didn’t do much, it just spit out 29 no matter what the person used as input. I moved on to book 705. The program in it was much more complicated. I made a few notes as I puzzled it out. “What do you think?” Merlin asked.

“This program doesn’t just talk about itself. It makes references to the programs written in other books. The algorithm assumes that this book is in spot 705, otherwise what it is doing wouldn’t work.”

“It won’t always be in spot 705,” Merlin noted, “but there will always be one just like it that is where it belongs. And since we know where to find it, we can always find a version of a program where the self reference works.”

“So self reference isn’t a problem when reading programs,” I noted.

“No, it isn’t” he replied. “But the best part is that it isn’t a problem when writing programs. It is perfectly legal to use the number assigned to your program without knowing how the programs will be ordered, even without knowing how you plan to end the program you are writing.”

“Wow, it’s magic!”

“Even better”, said Merlin, “it’s math.”