Section 9.2

The Euler Methods

EULER'S METHOD

One of the simplest techniques for approximating solutions of differential equations is **Euler's Method**, which is also known as the **method of tangent lines**.

PROCEDURE FOR EULER'S METHOD

1. Given $y' = f(x, y), y(x_0) = y_0$, find the slope of the tangent line at (x_0, y_0). The slope is $y'(x_0, y_0) = f(x_0, y_0)$. We denote this slope by y_0'.
2. Find a point $(x_1, y_1) = (x_0 + h, y_1)$ on the tangent line by the formula $y_1 = y_0 + hy_0'$. The variable h represents the **step size** which is "reasonably small."
3. Using the same value for h, we find the slope y_1' at (x_1, y_1). Find (x_2, y_2) by $x_2 = x_1 + h$ and $y_2 = y_1 + hy_1'$.

PROCEDURE (CONCLUDED)

4. By continuing in the above manner, we are able to draw an approximate solution curve.

NOTE: In general,

$$y_{n+1} = y_n + hy_n' = y_n + hf(x_n, y_n)$$

where $x_n = x_0 + nh$.

ERROR

Absolute Error: |true value − approximation|

Relative Error: $\dfrac{|\text{true value} - \text{approximation}|}{|\text{true value}|} = \dfrac{\text{absolute error}}{|\text{true value}|}$

Percentage Relative Error:

$$\frac{|\text{true value} - \text{approximation}|}{|\text{true value}|} \times 100 = \frac{\text{absolute error}}{|\text{true value}|} \times 100 = (\text{relative error}) \times 100$$

IMPROVED EULER'S METHOD

This method is the same as Euler's method except that it uses a more accurate approximations. It uses the **improved Euler's formula**, or **Heun's formula**.

$$y_{n+1} = y_n + h\frac{f(x_n, y_n) + f(x_{n+1}, y_{n+1}^*)}{2},$$

where $y_{n+1}^* = y_n + hf(x_n, y_n)$.