Section 9.2

The Euler Methods

EULER'S METHOD

One of the simplest techniques for approximating solutions of differential equations is <u>Euler's Method</u>, which is also known as the <u>method of tangent lines</u>.

PROCEDURE FOR EULER'S METHOD

- 1. Given $y' = f(x, y), y(x_0) = y_0$, find the slope of the tangent line at (x_0, y_0) . The slope is $y'(x_0, y_0) = f(x_0, y_0)$. We denote this slope by y'_0 .
- 2. Find a point $(x_1, y_1) = (x_0 + h, y_1)$ on the tangent line by the formula $y_1 = y_0 + hy'_0$. The variable *h* represents the <u>step size</u> which is "reasonably small."
- 3. Using the same value for *h*, we find the slope y'_1 at (x_1, y_1) . Find (x_2, y_2) by $x_2 = x_1 + h$ and $y_2 = y_1 + hy'_1$.

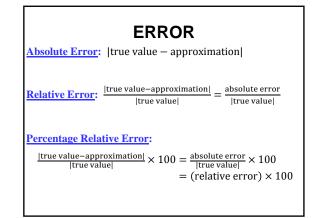
PROCEDURE (CONCLUDED)

4. By continuing in the above manner, we are able to draw an approximate solution curve.

NOTE: In general,

 $y_{n+1} = y_n + hy'_n = y_n + hf(x_n, y_n)$

where $x_n = x_0 + nh$.



IMPROVED EULER'S METHOD

This method is the same as Euler's method except that it uses a more accurate approximations. It uses the <u>improved Euler's</u> <u>formula</u>, or <u>Heun's formula</u>.

$$y_{n+1} = y_n + h \frac{f(x_n, y_n) + f(x_{n+1}, y_{n+1}^*)}{2},$$

where
$$y_{n+1}^* = y_n + hf(x_n, y_n)$$
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