Section 8.2

Laplace Transform Method [for Solving a System of Linear Differential Equations]

SYSTEMS OF DIFFERENTIAL EQUATIONS

Simultaneous ordinary differential equations involve two or more equations that contain derivatives of two or more unknown functions of a single independent variable. If x, y, and z are functions of the variable t, then two examples of systems of simultaneous differential equations are

$$4\frac{d^2x}{dt^2} = -5x + y \qquad 2\frac{d^2y}{dt^2} = 3x - y$$

and

$$x' - 3x + y' + z' = 5 \qquad x' - y' + 2z' = t^2 \qquad x + y' - 6z' = t - 1$$

SOLUTION OF A SYSTEM

A **solution** to a system of differential equations is a set of differentiable functions $x(t) = f(t), y(t) = g(t), z(t) = h(t)$ and so on, that satisfies each equation on some interval I.

LAPLACE TRANSFORMS AND SYSTEMS OF DIFFERENTIAL EQUATIONS

A system of first-order differential equations can be solved using Laplace transforms ***as long as initial conditions are given***.

LAPLACE TRANSFORM METHOD

1. Find the Laplace transform of each equation. This gives a system of equations in $X(s), Y(s)$, and so on.
2. Algebraically solve the system from Step 1 for $X(s), Y(s)$, and so on.
3. Recover $x(t), y(t)$, and so on by taking the inverse Laplace transform of $X(s), Y(s)$, and so on, respectively.