Section 8.1

Operator Method

DIFFERENTIAL OPERATOR

In calculus, differentiation is often denoted by the capital letter D; that is,

$$\frac{dy}{dx} = Dy$$

The symbol D is called a **differential operator**, it transforms a differentiable function into another function.

The differential operator is a **linear** operator.

HIGHER-ORDER DERIVATIVES

Higher order derivatives can be expressed in terms of the differential operator.

$$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = D(Dy) = D^2y$$

In general,

$$y^{(n)} = \frac{d^ny}{dx^n} = D^ny.$$

POLYNOMIAL EXPRESSIONS AND DIFFERENTIAL OPERATORS

Polynomial expressions involving D are also linear differential operators.

EXAMPLES:

$D + 3$
$D^2 + 3D - 4$
$5D^3 - 6D^2 + 4D + 9$

WRITING A DIFFERENTIAL EQUATION IN OPERATOR NOTATION

Differential equations can be written in operator notation.

SYSTEMS OF DIFFERENTIAL EQUATIONS

Simultaneous ordinary differential equations involve two or more equations that contain derivatives of two or more unknown functions of a single independent variable. If x, y, and z are functions of the variable t, then two examples of systems of simultaneous differential equations are

$$\begin{aligned} 4\frac{d^2x}{dt^2} &= -5x + y \\ 2\frac{d^2y}{dt^2} &= 3x - y \end{aligned} \qquad \text{and} \qquad \begin{aligned} x' - 3x + y' + z' &= 5 \\ x' \quad - y' + 2z' &= t^2 \\ x + y' - 6z' &= t - 1 \end{aligned}$$

SOLUTION OF A SYSTEM

A **solution** to a system of differential equations is a set of differentiable functions $x(t) = f(t), y(t) = g(t), z(t) = h(t)$ and so on, that satisfies each equation on some interval I.

SOLVING A SYSTEM BY SYSTEMATIC ELIMINATION

1. Add appropriate multiples of each equation together to eliminate the function $y(t)$.
2. Solve for the function $x(t)$.
3. Add appropriate multiples of each equation together to eliminate the function $x(t)$.
4. Solve for the function $y(t)$.
5. Substitute $x(t)$ and $y(t)$ (and their derivatives) into one or more of the equations to determine the "values" for the constants.