Section 7.5

Applications

USING THE LAPLACE TRANSFORM TO SOLVE DE'S

1. Transform the equation using the Laplace transforms. Note that this results in an equation that can be solved ***algebraically***.
2. Solve the transformed equation.
3. Use the inverse Laplace transform to find the solution to the original equation.

EXAMPLES

1. $\frac{dy}{dt} + 2y = t, y(0) = -1$
2. $y'' - 4y' + 4y = t^3, y(0) = 1, y'(0) = 0$
3. $y' + y = f(t)$, where $f(t) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & t \ge 0 \end{cases}, y(0) = 0$

VOLTERRA INTEGRAL EQUATION

The Convolution Theorem can be used to solve equations in which an unknown function appears under an integral sign. An example is the **Volterra integral equation**

$$f(t) = g(t) + \int_0^t f(\tau)h(t-\tau)d\tau$$

where $g(t)$ and $h(t)$ are known.

EXAMPLE

Solve for f using the Laplace transform.

$$f(t) = 2t - \int_0^t f(\tau)\sin(t-\tau)d\tau$$