Section 7.2

Inverse Transform

INVERSE LAPLACE TRANSFORM

In the last section we defined the Laplace transform of a function $f(t)$ and denoted it by $\mathcal{L}\{f(t)\} = F(s)$. We want turn the question around and ask if we can find $f(t)$ given $F(s)$. We can, although the mechanics are beyond the scope of this course. We call the new transform the **inverse Laplace transform**. It is denoted by

$$f(t) = \mathcal{L}^{-1}\{F(s)\}.$$

NOTE: The inverse Laplace transform is linear.

SOME INVERSE TRANSFORMS

1. $1 = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$
2. $t^n = \mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\}, n = 1, 2, 3, \ldots$
3. $e^{at} = \mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\}$
4. $t^n e^{at} = \mathcal{L}^{-1}\left\{\frac{n!}{(s-a)^{n+1}}\right\}, n = 1, 2, 3, \ldots$
5. $\sin kt = \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\}$
6. $\cos kt = \mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\}$

PARTIAL FRACTIONS

To find the inverse Laplace transformation, many times partial fraction decomposition is needed. We will work with

(i) only distinct linear factors

(ii) repeated linear factors

(iii) irreducible quadratic factors

and combinations of these types.

NOTE: These are the same types studied in Calculus II.

BEHAVIOR OF $F(s)$ AS $s \to \infty$

Theorem: Let $f(t)$ be piecewise continuous on $[0, \infty)$ and of exponential order for $t > T$; then

$$\lim_{s\to\infty} F(s) = \lim_{s\to\infty} \mathcal{L}\{f(t)\} = 0.$$

NOTE: This theorem is used to determine if a functions of s is not a piecewise Laplace transform of exponential order. If the $\lim_{s\to\infty} F(s) \neq 0$, then $F(s)$ is not the Laplace transform of a piecewise continuous function of exponential order.