Section 6.2

Review of Power Series;
Power Series Solutions

REVIEW OF POWER SERIES

1. An infinite series $P(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ is called a **power series** in $(x-a)$ or a **power centered at a**.
2. $P(x)$ is **convergent** at a point $x = c$ if $P(c) < \infty$. If $P(c) = \pm\infty$, the series **diverges** at c.
3. The set of all numbers for which a series converges is called its **interval of convergence**. One-half the length of this interval is its **radius** (R) **of convergence**.

POWER SERIES (CONTINUED)

4. Exactly ***one*** of the following is true of a power series.

   (a) the series converges only at center a; $R = 0$.

   (b) the series converges for all x in $|x-a| < R$, and diverges for all x in $|x-a| > R$, $R > 0$.

   (c) the series converges for all x; $R = \infty$.
5. A series may or may not converge at an endpoint of its interval of convergence.

POWER SERIES (CONTINUED)

6. The **ratio test**

$$\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right||x-a| = L$$

can be used to determine the interval of convergence. The series **converges absolutely** for all x for which $L < 1$. The radius of convergence is

$$R = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$$

if the limit exists.

POWER SERIES (CONTINUED)

7. Power series can be combined through the operations of addition, subtractions, multiplication, and division.
8. If $P(x)$ has a radius of convergence $R > 0$, then it is continuous and can be differentiated or integrated term by term on $(a-R, a+R)$. However, the resulting series may or may not be convergent at an endpoint of $(a-R, a+R)$.

POWER SERIES (CONCLUDED)

9. A function f is **analytic at a point a** if it can be represented by a power series in $(x-a)$ with $R > 0$.

POWER SERIES SOLUTION TO A DIFFERENTIAL EQUATION

To find a power series solution to a differential equation:

1. Assume a series exists in the form $\sum_{n=0}^{\infty} c_n (x-a)^n$
2. Take derivatives and substitute the series into the differential equation.
3. Solve a recurrence relation for the constants, c_n.
4. Write the power series solution.