Section 5.3

Forced Motion

FORCED MOTION

Suppose we take into account an external force $f(t)$ acting on a vibrating mass on a spring. Including $f(t)$ in the formulation of Newton's second law gives the differential equation of ***forced motion***.

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt} + f(t)$$

$$\frac{d^2t}{dx^2} + \frac{\beta}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{f(t)}{m} \text{ or } \frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = F(t)$$

where $F(t) = \frac{f(t)}{m}$, $2\lambda = \frac{\beta}{m}$, and $\omega^2 = \frac{k}{m}$.

SOLVING A FORCED MOTION DE

There is no general solution for the DE on the previous slide. It is a nonhomogeneous DE and will have a complimentary solution $x_c(t)$ and a particular solution $x_p(t)$. The particular solution depends upon the function $F(t)$. The complementary solution will be the solution to

- simple harmonic motion (Section 5.1) if the system is *undamped*, and
- damped motion (Section 5.2) if the system is *damped*.

TRANSIENT SOLUTION AND STEADY-STATE SOLUTION

If $x_c(t) \to 0$ as $t \to \infty$, it is said to be a ***transient term***, or ***transient solution***. So, for large values of t, the displacement of the weight is closely approximated by the particular solution $x_p(t)$. The particular solution is also called the ***steady-state solution***. Transient and steady-state solutions only occur with *damped motion*.