Section 5.1

Simple Harmonic Motion

SIMPLE HARMONIC MOTION

The second-order differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \text{ or } \frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega^2 = k/m$ is the equation that describes **simple harmonic motion**, or **free undamped motion**.

INITIAL CONDITIONS

The initial conditions for simple harmonic motion are:

$$x(0) = \alpha, x'(0) = \beta$$

NOTES:

1. If $\alpha > 0, \beta < 0$, the mass starts from a point $|\alpha|$ units ***below*** the equilibrium position with an imparted ***upward*** velocity.
2. If $\alpha < 0, \beta = 0$, the mass is released from *rest* from a point $|\alpha|$ units ***above*** the equilibrium position.

SOLUTION

The general solution of the equation for simple harmonic motion is

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t$$

The **period** of the free vibrations is $T = \frac{2\pi}{\omega}$, and the **frequency** of the vibrations is $f = \frac{1}{T} = \frac{\omega}{2\pi}$.

ALTERNATIVE FORM OF *x*(*t*)

When $c_1 \neq 0$ and $c_2 \neq 0$, the actual **amplitude** A of the vibrations is not obvious from the equation on the previous slide. We often convert the equation to the simpler form

$$x(t) = A \sin(\omega t + \phi),$$

where $A = \sqrt{c_1^2 + c_2^2}$ and ϕ is a **phase angle** defined by

$$\left.\begin{aligned} \sin \phi &= \frac{c_1}{A} \\ \cos \phi &= \frac{c_2}{A} \end{aligned}\right\} \text{ which is equivalent to } \tan \phi = \frac{c_1}{c_2}$$