Section 4.6

Undetermined Coefficients—Annihilator Approach

UNDETERMINED COEFFICIENTS—ANNHILATOR APPROACH

The differential equation $L(y) = g(x)$ has constant coefficients and the function $g(x)$ consists of finite sums and products of polynomials, exponential functions $e^{\alpha x}$, sines, and cosines.

1. Find the complementary solution y_c for the homogeneous equation $L(y) = 0$.
2. Operate on both sides of the nonhomogeneous equation $L(y) = g(x)$ with a differential operator L_1 that annihilates the function $g(x)$.

ANNIHILATOR APPROACH (CONCLUDED)

3. Find the general solution to the higher-order homogeneous differential equation $L_1L(y) = 0$.
4. Delete all those terms from the solution in Step 3 that are duplicated in the complementary solution y_c. Form a linear combination of the terms that remain. This is the form of the particular solution of $L(y) = g(x)$.
5. Substitute y_p found in Step 4 into $L(y) = g(x)$. Match the coefficients of the various functions on each side of the equality and solve the resulting system of equations for the unknown coefficients in y_p.
6. Write the general solution: $y = y_c + y_p$.