

Section 4.2

Constructing a Second Solution from a Known Solution

REDUCTION OF ORDER

Suppose $y_1(x)$ is a nonzero solution of the equation

$$a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y = 0.$$

The process we use to find a second solution $y_2(x)$ consists of **reducing the order** of the above equation to a first-order equation.

SOLUTION TO A LINEAR 2ND- ORDER HOMOGENEOUS DE

The standard form for a linear second-order homogeneous differential equation is

$$y'' + P(x)y' + Q(x)y = 0.$$

Given a solution $y_1(x)$, a second solution to the equation is

$$y_2 = y_1(x) \int \frac{e^{\int P(x)dx}}{y_1^2} dx.$$

NOTE: The solutions y_1 and y_2 are linearly independent.