Section 3.3

Applications of Nonlinear Equations

THE LOGISTIC EQUATION

The equation

$$\frac{dP}{dt} = P(a - bP),$$

where a and b are constants, is called the **logistic equation**. Its solution is called the **logistic function** (the graph of which is called the logistic curve).

EXAMPLE

The number of people in a community who are exposed to a particular advertisement is governed by the logistic equation. Initially $N(0) = 500$, and it is observed that $N(1) = 1000$. If it is predicted that the limiting number of people in the community who will see the advertisement is 50,000, determine $N(t)$ at time t.

SECOND ORDER CHEMICAL REACTIONS

Radioactive decay, where the rate at which decomposition takes place is proportional to the amount present, is said to be a **first-order reaction**. Now in the reaction

$$CH_3Cl + NaOH \rightarrow CH_3OH + NaCl$$

the rate at which the reaction proceeds depends on both the remaining amount of CH_3Cl and the remaining NaOH. This is an example of a **second-order reaction**. A differential equation for this is given by

$$\frac{dX}{dt} = k(\alpha - X)(\beta - X)$$

where α and β are the given amounts of CH_3Cl and NaOH and X is the amount of CH_3OH produced.

EXAMPLE

A compound C is formed when two chemicals A and B are combined. The resulting reaction between the two chemicals is such that for each gram of A, 3 grams of B are used. It is observed that 30 grams of compound C are formed in 10 minutes. Determine the amount of C at any time if the rate of reaction is proportional to the amounts of A and B remaining and if initially there are 40 grams of A and 27 grams of B. How much of the compound C is present at 20 minutes? Interpret the solution as $t \to \infty$.

ESCAPE VELOCITY

In Section 1.2, we saw that the differential equation of a free-falling object of mass m near the surface of the earth is

$$m\frac{d^2s}{dt^2} = -mg \quad \text{or simply} \quad \frac{d^2s}{dt^2} = -g$$

where s represents the distance from the surface of the earth. The assumption is that the distance y from the center of the earth is approximately the radius R of the earth. If we consider a rocket (space probe, etc.) whose distance y is large when compared to R, we combine Newton's second law of motion and his law of universal gravitation to produce a differential equation in the variable y.

ESCAPE VELOCITY (CONCLUDED)

The solution to the differential equation can be used to determine the minimum velocity needed by a rocket to break free from the earth's gravitational attraction. This velocity is called the **escape velocity**.