

Section 3.2

Applications of Linear Equations

GROWTH AND DECAY

In previous mathematics courses you have studied growth and decay problems. Some examples are population growth, radioactive decay, and carbon dating.

The differential equation for growth and decay is

$$\frac{dx}{dt} = kx, \quad x(t_0) = x_0$$

If $k > 0$, it models growth. If $k < 0$, it models decay. We will not discuss these applications in class. Rather, you will have some homework and assignments reviewing them.

NEWTON'S LAW OF COOLING

Recall from Chapter 1 that Newton's Law of cooling states that the rate at which an object cools is proportional to the difference in the temperature of the object and the surrounding medium. That is,

$$\frac{dT}{dt} = k(T - T_m), \quad T(t_0) = T_0$$

EXAMPLE

Newton's law of cooling can be used to estimate the time of death. Suppose the temperature of a victim of a homicide is 85° when it is discovered and the ambient temperature is 68° . If after two hours the corpse has a temperature of 74° , determine the time of death. (Assume the ambient temperature remains the same.)

SERIES CIRCUITS

In a series circuit containing only a resistor and an inductor, Kirchhoff's second law states that the sum of the voltage drop across the inductor, $L \frac{di}{dt}$, and the voltage drop across the resistor, iR , is the same as the impressed voltage, $E(t)$, on the circuit. Thus, the linear DE for the current $i(t)$ is

$$L \frac{di}{dt} + Ri = E(t)$$

where L and R are constants known as the inductance and resistance, respectively. The current $i(t)$ is called the **response** of the system.

SERIES CIRCUITS AND CAPACITANCE

The voltage drop across a capacitor with capacitance C is given by $\frac{q(t)}{C}$, where q is the charge on the capacitor.

Hence for a series circuit with a capacitor, Kirchhoff's second law gives

$$Ri + \frac{1}{C}q = E(t).$$

But current i and charge q are related by $i = \frac{dq}{dt}$, so the equation above yields the linear differential equation

$$R \frac{dq}{dt} + \frac{1}{C}q = E(t)$$

EXAMPLE

A 200-volt electromotive force is applied to an R - C series circuit in which the resistance is 1000 ohms and the capacitance is 5×10^{-6} farad. Find the charge $q(t)$ on the capacitor if $i(0) = 0.4$. Determine the charge and current at $t = 0.005$ second. Determine the charge as $t \rightarrow \infty$.

NOTE: The term in the equation that approaches 0 as $t \rightarrow \infty$, is called a **transient term** and any remaining terms are called the **steady-state** part of a solution.

A MIXTURE PROBLEM

At time $t = 0$ a tank contains 15 pounds of salt dissolved in 100 gallons of water. Assume that water containing $\frac{1}{4}$ pound of salt per gallon is entering the tank at the rate of 3 gallons per minute, and that the well-stirred solution is leaving the tank at the same rate. Find an expression for the amount of salt in the tank at time t . How much salt is present after 45 minutes? after a long time?
