Section 3.1

Orthogonal Trajectories

DIFFERENTIAL EQUATION FROM AN *n*-PARAMETER FAMILY

In Chapter 1, we discussed that the solution to an n^{th}-order differential equation is an n-parameter family. Now, we ask the question: Can an n-parameter family (with n arbitrary constants) produce an n^{th}-order differential equation ***without*** any constants?

YES!!!!

In this section, we are only going to look at 1-parameter families that will produce first-order differential equations.

ORTHOGONAL CURVES

Two curves (graphs) are said to be **orthogonal** at a point if their tangents lines are perpendicular at that point. That is, the slopes of the two tangent lines are negative reciprocals of each other.

ORTHOGONAL TRAJECTORIES

When all the curves of one family of curves $G(x, y, c_1) = 0$intersect orthogonally all the curves of another family $H(x, y, c_2) = 0$, then the families are said to be **orthogonal trajectories** of each other.

NOTE: Orthogonal trajectories occur naturally in the construction of meteorological maps and in the study of electricity and magnetism.

FINDING ORTHOGONAL TRAJECTORIES

To find the orthogonal trajectory of a given family of curves

1. Find the differential equation that describes the family.

$$\frac{dy}{dx} = f(x, y)$$

2. The differential equation of the second, and orthogonal, family is then

$$\frac{dy}{dx} = -\frac{1}{f(x, y)}$$

3. Solve the equation in Step 2.