Section 2.6

Equations of Bernoulli, Ricatti, and Clairaut

BERNOULLI'S EQUATION

The differential equation

$$\frac{dy}{dx} + P(x)y = f(x)y^n,$$

where n is any real number, is called **Bernoulli's equation**. For $n = 0$ and $n = 1$, the equation is linear and can be solved by methods of the last section.

SOLVING A BERNOULLI EQUATION

1. Rewrite the equation as
$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = f(x)$$
2. Use the substitution $w = y^{1-n}, n \neq 0, n \neq 1$.
NOTE: $\frac{dw}{dx} = (1-n)y^{-n}\frac{dy}{dx}$.
3. This substitution turns the equation in Step 1 into a linear equation which can be solved by the method of the last section.

RICATTI'S EQUATION

The nonlinear differential equation

$$\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2$$

is called **Ricatti's equation**.

SOLVING A RICATTI EQUATION

1. Find a particular solution y_1. (This may be given.)
2. Use the substitution $y = y_1 + u$ and
$$\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{du}{dx}$$
to reduce the equation to a Bernoulli equation with $n = 2$.
3. Solve the Bernoulli equation.

CLAIRAUT'S EQUATION

The nonlinear differential equation

$$y = xy' + f(y')$$

is called **Clairaut's equation**. Its solution is the family of straight lines $y = cx + f(c)$, where c is an arbitrary constant. (See Problem 29.)

PARAMETRIC SOLUTION TO CLAIRAUT'S EQUATION

Clairaut's equation may also have a solution in parametric form:

$$x = -f'(t), y = f(t) - t\,f'(t).$$

NOTE: This solution will be a singular solution since, if $f''(t) \neq 0$, it cannot be determined from the family of solutions $y = cx + f(c)$.