

Section 2.5

Linear Equations

LINEAR FIRST-ORDER EQUATION

Definition: A first-order differential equation of the form

$$a_1(x) \frac{dy}{dx} + a_0(x)y = g(x)$$

is said to be a **linear first-order equation**. The standard form for a linear first-order differential equation is

$$\frac{dy}{dx} + P(x)y = f(x)$$

SOLVING A LINEAR FIRST-ORDER DE

To solve $\frac{dy}{dx} + P(x)y = f(x)$

1. Multiply both sides by the integrating factor

$$e^{\int P(x)dx}$$

2. Write the result in the form

$$\frac{d}{dx} [e^{\int P(x)dx} y] = e^{\int P(x)dx} f(x)$$

3. Integrate both sides to solve for y .

GENERAL SOLUTION

The procedure on the previous slide produces the **general solution** to a linear first-order differential equation. For an initial-value problem, we use the general solution to obtain a **particular solution** to the equation.

NOTE: A linear first-order differential equation **CANNOT** have a singular solution.