Section 2.3

Homogeneous Equations

HOMOGENEOUS FUNCTION

Definition: A function, $f(x, y)$, is said to be **homogeneous of degree n** if

$$f(tx, ty) = t^n f(x, y)$$

for some real number n.

EXAMPLES

Determine whether the function is homogeneous. If so, state the degree of homogeneity.

1. $f(x, y) = x^2y - 4x^3 + 3xy^2$
2. $f(x, y) = x + y^2$
3. $f(x, y) = \frac{x}{y}$

HOMOGENEOUS DIFFERENTIAL EQUATION

Definition: A differential equation of the form

$$M(x, y)dx + N(x, y)dy = 0$$

is said to be **homogeneous** if both M and N are homogeneous functions of the same degree.

EXAMPLE: $(x^2 + y^2)dx + 2xy\,dy = 0$

ALTERNATE VIEW OF HOMOGENEOUS EQUATION

The differential equation

$$\frac{dy}{dx} = f(x, y)$$

is a homogeneous equation if $f(x, y)$ depends on the ratio $\frac{x}{y}$ or $\frac{y}{x}$. That is, it can be written in the form

$$F\left(\frac{x}{y}\right) \text{ or } F\left(\frac{y}{x}\right)$$

for some function F.

EXAMPLE: $\frac{dy}{dx} = \frac{x}{2y} + \frac{y}{2x}$

SOLVING A HOMOGENEOUS EQUATION

1. Use the substitution $y = ux$ to change the dependent variable from y to u. Note that

$$\frac{dy}{dx} = \frac{du}{dx}x + u \quad \text{and}$$

$$dy = u\,dx + x\,du$$

2. Separate the variables and solve the equation using the techniques of Section 2.2.

NOTE: The substitution $x = vy$ can also be used. As a guide, use this one whenever $M(x, y)$ is simpler than $N(x, y)$.