

Section 2.1

Preliminary Theory

INITIAL-VALUE PROBLEM

Most first-order DEs can be written in the form

$$\frac{dy}{dx} = f(x, y) \quad (i)$$

When asked to solve (i) given the side condition that

$$y(x_0) = y_0 \quad (ii),$$

we call the problem, (i) and (ii), an **initial-value problem** (IVP). The side condition is known as an **initial condition**.

TWO QUESTIONS CONCERNING AN IVP

1. Does a solution to the problem *exist*?
2. If a solution exists, is it *unique*?

EXISTENCE OF A UNIQUE SOLUTION

Theorem: Let R be a rectangular region in the xy -plane defined by $a \leq x \leq b, c \leq y \leq d$ that contains the point (x_0, y_0) in its interior. If $f(x, y)$ and $\frac{\partial f}{\partial y}$ are continuous on R , then there exists an interval I centered at x_0 and a unique function $y(x)$ defined on I satisfying the initial-value problem.