Section 1.1

Basic Definitions and Terminology

DIFFERENTIAL EQUATIONS

Definition: A **differential equation** (DE) is an equation containing the derivatives or differentials of one or more dependent variables, with respect to one or more independent variables.

PARTIAL DERIVATIVES

If $z = f(x, y)$ is a function of two variables, its **partial derivatives** are the functions $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ defined by

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial z}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

RULE FOR FINDING PARTIAL DERIVATIVES OF $z = f(x, y)$

1. To find $\frac{\partial f}{\partial x}$, regard y as a constant and differentiate $f(x, y)$ with respect to x.
2. To find $\frac{\partial f}{\partial y}$, regard x as a constant and differentiate $f(x, y)$ with respect to y.

CLASSIFICATION OF DIFFERENTIAL EQUATIONS

Differential equations are classified according to

(i) type

(ii) order

(iii) linearity

CLASSIFICATION BY TYPE

Differential equations are divided into two *types*.

1. An equation involving only ordinary derivatives of one or more dependent variables of a ***single*** independent variable is called an **ordinary differential equation** (ODE).
2. An equation involving the partial derivatives of one or more dependent variables of ***two or more*** independent variables is called a **partial differential equation** (PDE).

CLASSIFICATION BY ORDER

The **order** of a differential equation is the order of the highest-order derivative in the equation.

CLASSIFICATION BY LINEARITY

Differential equations are classified by linearity as follows.

1. If the dependent variable (y) and its derivatives are of the first degree, and each coefficient depends only on the independent variable (x), then the differential equation is **linear**.
2. Otherwise, the differential equation is **nonlinear**.

LINEAR DIFFERENTIAL EQUATION

A differential equation is said to be **linear** if it can be written in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

SOLUTION OF A DIFFERENTIAL EQUATION

Definition: Any function *f* defined on some interval *I*, which when substituted into a differential equation reduces the equation to an identity, is said to be a **solution** of the equation on the interval *I*.

NOTE: Depending on the context of the problem the interval *I* could be an open interval, a closed interval, a half-open interval, or an infinite interval.

AN *n*-PARAMETER FAMILY OF SOLUTIONS

When solving an n^{th}-order differential equation $F\left(x, y, y', \ldots, y^{(n)}\right) = 0$, we expect a solution $G(x, y, c_1, \ldots, c_n) = 0$ with n arbitrary parameters (constants). Such a solution is called an ***n*-parameter family of solutions**.

PARTICULAR SOLUTIONS

A solution of a differential equation that is free of arbitrary parameters is called a **particular solution**. One way of obtaining a particular solution is to choose specific values of the parameter(s) in a family of solutions.

A particular solution that ***cannot*** be obtained by specializing the parameters in a family of solutions is called a **singular solution**.