

REVIEW FOR TEST II – IN-CLASS PORTION
MATH 3502

1. (a) Find the orthogonal trajectories of the family of curves $y(x^3 + c_1) = 3$.
(b) Find the orthogonal trajectory to the family $y = 4x + 1 + c_1 e^{4x}$ passing through the point $(0, 0)$.
(c) Find the orthogonal trajectories of the family of parabolas opening in the y direction with vertex at $(1, 2)$.

2. (a) The radioactive isotope plutonium-241 decays so as to satisfy the differential equation

$$\frac{dA}{dt} = -0.0525A,$$

Where A is measured in milligrams and t in years. Determine the half-life h of plutonium-241. If 50 mg of plutonium-241 are present today, how much will remain in 10 years?

- (b) An object is taken out of a 65°F room and placed outside where the temperature is 35°F . Fifteen minutes later the temperature is 60°F . It cools according to Newton's Law. What is the temperature of the object after one hour?
- (c) The population of a town grows at a rate proportional to the number of people, $P(t)$, at time t . If the initial population is P_0 , and if the size doubles in twelve years, how long does it take the population to triple in size?

3. Solve the following differential equations.

(a) $y'' + 2y' + y = 0$

(b) $y'' - 2y' + 6y = 0$

(c) $y'' - 2y' - 2y = 0$

(d) $y'' + 5y' = 0$

(e) $4y'' + 4y' + y = 0$

(f) $y'' + 6y' + 13y = 0$

(g) $y'' - 3y' + 5y = 4x^3 - 2x$

(h) $y'' - 2y' + y = x^2 e^x$

(i) $y''' - 5y'' + 6y' = 2 \sin x + 8$

(j) $y''' - y'' = 6$

ANSWERS

1. (a) $y^3 + \frac{3}{x} = c_2$

(b) $x = \frac{1}{4}y + \frac{1}{64} - \frac{1}{64}e^{16y}$

$$(c) \quad (y - 2)^2 = x - \frac{1}{2}x^2 + c_2$$

2. (a) $h = 13.20$ years; 29.6 mg (b) 49.5°F
 (c) 19.8 years

3. (a) $y = c_1 e^{-x} + c_2 x e^{-x}$ (b) $y = c_1 e^x \sin \sqrt{5}x + c_2 e^x \cos \sqrt{5}x$
 (c) $y = c_1 e^{(1+\sqrt{3})x} + c_2 e^{(1-\sqrt{3})x}$ (d) $y = c_1 + c_2 e^{-5x}$
 (e) $y = c_1 e^{-x/2} + c_2 x e^{-x/2}$ (f) $y = e^{-3x}(c_1 \cos 2x + c_2 \sin 2x)$
 (g) $y = e^{3x/2} \left(c_1 \cos \frac{\sqrt{11}}{2}x + c_2 \sin \frac{\sqrt{11}}{2}x \right) + \frac{4}{5}x^3 + \frac{36}{25}x^2 + \frac{46}{125}x - \frac{222}{625}$
 (h) $y = c_1 e^x + c_2 x e^x + \frac{1}{12}x^4 e^x$
 (i) $y = c_1 + c_2 e^{2x} + c_3 e^{3x} + \frac{4}{3}x - \frac{1}{5}\cos x + \frac{1}{5}\sin x$
 (j) $y = c_1 + c_2 x + c_3 e^x - 3x^2$