STUDY GUIDE FOR TEST I
MATH 3502

1. Classify the given differential equation as to type and order. Classify the ordinary differential equations as to linearity.

    (a) $(2xy - y^2)dx + e^x dy = 0$

    (b) $(\sin xy)y''' + 4xy' = 0$

    (c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$

    (d) $x^2\frac{d^2 y}{dx^2} - 3x\frac{dy}{dx} + y = x^2$

2. Verify that the indicated function is a solution of the given differential equation.

    (a) $y' + 2xy = 2 + x^2 + y^2$; $y = x + \tan x$

    (b) $x^2 y'' + xy' + y = 0$; $y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$, $x > 0$

    (c) $y''' - 2y'' - y' + 2y = 6$; $y = c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + 3$

3. Without solving classify each of the following equations as to whether it is separable, homogeneous, exact, linear, Bernoulli, Ricatti, or Clairaut.

    (a) $\frac{dy}{dx} = \frac{1}{y-x}$

    (b) $\frac{dy}{dx} = \frac{x-y}{x}$

    (c) $\left(\frac{dy}{dx}\right)^2 + 2y = 2x\frac{dy}{dx}$

    (d) $\frac{dy}{dx} = \frac{1}{x(x-y)}$

    (e) $\frac{dy}{dx} = \frac{y^2+y}{x^2+x}$

    (f) $\frac{dy}{dx} = 4 + 5y + y^2$

    (g) $y\,dx = (y - xy^2)dy$

    (h) $x\frac{dy}{dx} = ye^{x/y} - x$

    (i) $xyy' + y^2 = 2x$

    (j) $2xyy' + y^2 = 2x^2$

    (k) $y\,dx + x\,dy = 0$

(l) $\left(x^2 + \frac{2y}{x}\right)dx = (3 - \ln x^2)dy$

(m) $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} + 1$

(n) $\frac{y}{x^2}\frac{dy}{dx} + e^{2x^3+y^2} = 0$

(o) $y = xy' + (y' - 3)^2$

(p) $y' + 5y^2 = 3x^4 - 2xy$

4. Solve the following differential equations.

(a) $\frac{y}{x}\frac{dy}{dx} = \frac{e^x}{\ln y}$ subject to $y(1) = 1$

(b) $xyy' = 3y^2 + x^2$ subject to $y(-1) = 2$

(c) $(6x + 1)y^2\frac{dy}{dx} + 3x^2 + 2y^3 = 0$

(d) $ye^{xy}\frac{dx}{dy} + xe^{xy} = 12y^2$ subject to $y(0) = -1$

(e) $(x^2 + 4)\frac{dy}{dx} = 2x - 8xy$ subject to $y(0) = -1$

(f) $x\frac{dy}{dx} + 4y = x^4y^2$ subject to $y(1) = 1$

ANSWER

1. (a) first-order; ordinary; nonlinear because of y^2
   (b) third-order; ordinary; nonlinear because of $\sin xy$
   (c) second-order; partial
   (d) second-order; ordinary; linear

3. (a) linear in x
   (b) linear in y, homogeneous, exact
   (c) Clairaut   (d) Bernoulli in x
   (e) separable   (f) separable, Riccati
   (g) linear in x (h) homogeneous
   (i) Bernoulli

(j) homogeneous, exact, Bernoulli
(k) linear in x and y, exact, separable, homogeneous
(l) exact, linear in y (m) homogeneous
(n) separable   (o) Clairaut
(p) Ricatti

4. (a) separable; $2y^2 \ln|y| - y^2 = 4xe^x - 4e^x - 1$
   (b) homogeneous; $2y^2 + x^2 = 9x^6$
   (c) Bernoulli and exact (work it both ways); $(6x + 1)y^3 = -3x^3 + c$
   (d) exact; $e^{xy} = 4y^3 + 5$
   (e) separable and linear with integrating factor $(x^2 + 4)^4$;
   $$y = \frac{1}{4} - \frac{320}{(x^2+4)^4}$$
   (f) Bernoulli; $y = (x^4 - x^4 \ln x)^{-1}$