<u>Section 11.5</u>: Alternating Series

<u>Alternating Series</u>:

Definition: An ***<u>alternating series</u>*** is a series whose terms alternate signs. For example,

$$\sum_{n=1}^{\infty}(-1)^n a_n \quad \text{or} \quad \sum_{n=1}^{\infty}(-1)^{n-1} a_n$$

Alternating Series Test (AST): If the alternating series

$$a_1 - a_2 + a_3 - a_4 + \cdots$$
or
$$-a_1 + a_2 - a_3 + a_4 - \cdots$$

satisfies

(i) $a_{n+1} \leq a_n$ for all n; that is, $\{a_n\}$ is a decreasing sequence, and

(ii) $\lim_{n\to\infty} a_n = 0$

then the series is convergent.

<u>NOTES</u>:

(a) The terms of $\{a_n\}$ must be decreasing ($a_{n+1} \leq a_n$) and $\lim_{n\to\infty} a_n = 0$. If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges by the <u>n^{th}-Term Test</u>.

(b) If the series is not an alternating series, $\lim_{n\to\infty} a_n = 0$ does ***<u>not</u>*** insure convergence!

<u>Examples</u>:

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n!}$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{2n-1}$

Approximating the Sum of an Alternating Series

Theorem: If a convergent alternating series satisfies the condition $a_{n+1} \leq a_n$, then the absolute value of the remainder R_N involved in approximating the sum s by s_N is less than (or equal to) the first neglected term. That is,

$$|s - s_N| = |R_N| \leq a_{N+1}\ .$$

Example: Approximate the sum of the following series by its first six terms. Find a bound for the error in your approximation.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$$