Section 7.2

Orthogonal Diagonalization

ORTHOGONALLY DIAGONALIZABLE MATRICES

If A and B are square matrices, then we say that A and B are **orthogonally similar** if there is an orthogonal matrix P such that $P^TAP = B$.

If A is orthogonally similar to some diagonal matrix, say

$$P^TAP = D$$

then we say that A is **orthogonally diagonalizable** and that P **orthogonally diagonalizes** A.

CONDITIONS FOR ORTHOGONAL DIAGONALIZABILITY

Theorem 7.2.1: If A is an $n \times n$ matrix, then the following are equivalent.

(a) A is orthogonally diagonalizable.

(b) A has an orthonormal set of n eigenvectors.

(c) A is symmetric.

SYMMETRIC MATRICES

Theorem 7.2.2: If A is a symmetric matrix, then:

(a) The eigenvalues of A are real numbers.

(b) Eigenvectors from different eigenspaces are orthogonal.

DIAGONALIZATION OF SYMMETRIC MATRICES

Step 1: Find a basis for each eigenspace of A.

Step 2: Apply the Gram-Schmidt process to each of these bases to obtain an orthonormal basis for each eigenspace.

Step 3: Form the matrix P whose columns are the basis vectors constructed in Step 2. This matrix will orthogonally diagonalize A, and the eigenvectors on the diagonal of $D = P^TAP$ will be in the same order as their corresponding eigenvectors in P.