Section 4.4

Coordinates and Basis

BASIS

If V is any vector space and $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a finite set of vectors in V, then S is called a **basis** for V if the following two conditions hold:

(a) S is linearly independent.

(b) S spans V.

UNIQUENESS OF A BASIS REPRESENTATION

Theorem 4.4.1: If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for a vector space V, then every vector $\mathbf{v}$ in V can be expressed in the form

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_n\mathbf{v}_n$$

in exactly one way.

COORDINATES RELATIVE TO A BASIS

If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for a vector space V, and if

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_n\mathbf{v}_n$$

is the expression for $\mathbf{v}$ in terms of the basis S, then the scalars $c_1, c_2, \ldots, c_n$ are the **coordinates** of $\mathbf{v}$ relative to S. The vector $(c_1, c_2, \ldots, c_n)$ in R^n constructed from these coordinates is called the **coordinate vector of v relative to *S***; it is denoted by

$$(\mathbf{v})_S = (c_1, c_2, \ldots, c_n).$$