Section 3.4

The Geometry of Linear Systems

VECTOR EQUATION OF A LINE

<u>Theorem 3.4.1</u>: Let L be a line in R^2 or R^3 that contains the point $\mathbf{x}_0$ and is parallel to the vector $\mathbf{v}$. Then the equation of the line through $\mathbf{x}_0$ that is parallel to $\mathbf{v}$ is

$$\mathbf{x} = \mathbf{x}_0 + t\mathbf{v}$$

If $\mathbf{x}_0 = \mathbf{0}$, then the line passes through the origin and the equation has the form

$$\mathbf{x} = t\mathbf{v}$$

VECTOR EQUATION OF A PLANE

<u>Theorem 3.4.2</u>: Let W be a plane in R^3 that contains the point $\mathbf{x}_0$ and is parallel to the noncollinear vectors $\mathbf{v}_1$ and $\mathbf{v}_2$. Then the equation of the plane through $\mathbf{x}_0$ that is parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$ is

$$\mathbf{x} = \mathbf{x}_0 + t_1\mathbf{v}_1 + t_2\mathbf{v}_2$$

If $\mathbf{x}_0 = \mathbf{0}$, then the plane passes through the origin and the equation has the form

$$\mathbf{x} = t_1\mathbf{v}_1 + t_2\mathbf{v}_2$$

LINES IN R^n

If $\mathbf{x}_0$ and $\mathbf{v}$ are vectors in R^n, and if $\mathbf{v}$ is nonzero, then the equation

$$\mathbf{x} = \mathbf{x}_0 + t\mathbf{v}$$

defines the **<u>line through $\mathbf{x}_0$ that is parallel to $\mathbf{v}$</u>**. In the special case where $\mathbf{x}_0 = \mathbf{0}$, the line is said to **<u>pass through the origin</u>**.

PLANES IN R^n

If $\mathbf{x}_0$ and $\mathbf{v}_1$ and $\mathbf{v}_2$ are vectors in R^n, and if $\mathbf{v}_1$ and $\mathbf{v}_2$ are not colinear, then the equation

$$\mathbf{x} = \mathbf{x}_0 + t_1\mathbf{v}_1 + t_2\mathbf{v}_2$$

defines the **<u>plane through $\mathbf{x}_0$ that is parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$</u>**. In the special case where $\mathbf{x}_0 = \mathbf{0}$, the plane is said to **<u>pass through the origin</u>**.

LINE SEGMENTS IN R^n

If $\mathbf{x}_0$ and $\mathbf{x}_1$ are vectors in R^n, then the equation

$$\mathbf{x} = \mathbf{x}_0 + t(\mathbf{x}_1 - \mathbf{x}_0) \quad (0 \le t \le 1)$$

defines the **<u>line segment from $\mathbf{x}_0$ to $\mathbf{x}_1$</u>**. When convenient, this equation can be written as

$$\mathbf{x} = (1 - t)\mathbf{x}_0 + t\mathbf{x}_1 \quad (0 \le t \le 1)$$

ORTHOGONALITY AND LINEAR SYSTEMS

<u>Theorem 3.4.3</u>: If A is an $m \times n$ matrix, then the solution set of the homogeneous linear systesm $A\mathbf{x} = \mathbf{0}$ consists of all vectors in R^n that are orthogonal to every row vector in A.

SOLUTION TO A CONSISTENT NONHOMEGENEOUS SYSTEM

<u>Theorem 3.4.4</u>: The general solution of a consistent linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding any specific solution of $A\mathbf{x} = \mathbf{b}$ to the general solution of $A\mathbf{x} = \mathbf{0}$.