Section 2.2

Evaluating Determinants by Row Reduction

A THEOREM

Theorem 2.2.1: Let A be a square matrix. If A has a row or column of all zeros, then $\det(A) = 0$.

TRANSPOSES AND THE DETERMINANT

Theorem 2.2.2: Let A be a square matrix. Then $\det(A) = \det(A^T)$.

NOTE: As a result of this theorem, nearly every theorem about the determinants that contains the word *row* in its statement is also true when the word *column* is substituted for *row*.

ROW OPERATIONS AND DETERMINANTS

Theorem 2.2.3: Let A be and $n \times n$ matrix.

(a) If B is the matrix that results when a single row or single column of A is multiplied by any scalar k, then $\det(B) = k \det(A)$.

(b) If B is the matrix that results when two rows or two columns of A are interchanged, then $\det(B) = -\det(A)$.

(c) If B is the matrix that results when a multiple of one row of A is added to another row or when a multiple of one column is added to another column, then $\det(B) = \det(A)$.

ELEMENTARY MATRICES AND DETERMINANTS

Theorem 2.2.4: Let E be an $n \times n$ elementary matrix.

(a) If E results from multiplying a row of I_n by k, then $\det(E) = k$.

(b) If E results from interchanging two rows of I_n, then $\det(E) = -1$.

(c) If E results from adding a multiple of one row of I_n to another, then $\det(E) = 1$.

DETERMINANTS AND PROPORTIONAL ROWS/COLUMNS

Theorem 2.2.5: If A is a square matrix with two proportional rows or two proportional columns, then $\det(A) = 0$.