# Section 1.7

**Diagonal, Triangular, and Symmetric Matrices**

## DIAGONAL MATRICES

A square matrix which consists of all zeros off the main diagonal is called a **diagonal matrix**.

Example:
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

## TRIANGULAR MATRICES

- A square matrix with all entries above the main diagonal zero is called a **lower triangular matrix**.
- A square matrix with all entries below the main diagonal zero is called an **upper triangular matrix**.
- A matrix that is either upper triangular or lower triangular is called **triangular**.

## EXAMPLES

$$\text{Lower Triangular}: \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\text{Upper Triangular}: \begin{bmatrix} -5 & 6 & 0 & 7 \\ 0 & 3 & -2 & -1 \\ 0 & 0 & \frac{2}{3} & 4 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

## THEOREM 1.7.1: PROPERTIES OF TRIANGULAR MATRICES

(a) The transpose of an upper triangular matrix is a lower triangular matrix, and the transpose of a lower triangular matrix is an upper triangular matrix.
(b) The product of lower triangular matrices is lower triangular; the product of upper triangular matrices is upper triangular.
(c) A triangular matrix is invertible if and only if its diagonal entries are all nonzero.
(d) The inverse of an invertible lower triangular matrix is lower triangular, and the inverse of an invertible upper triangular matrix is upper triangular.

## SYMMETRIC MATRICES

A square matrix is called **symmetric** if $A = A^T$.

## PROPERTIES OF SYMMETRIC MATRICES

**Theorem 1.7.2:** If $A$ and $B$ are symmetric matrices and if $k$ is a scalar, then:

(a) $A^T$ is symmetric.

(b) $A + B$ and $A - B$ are both symmetric.

(c) $kA$ is symmetric.

## PRODUCT OF SYMMETRIC MATRICES

**Theorem 1.7.3:** The product of two symmetric matrices is symmetric if and only if the matrices commute.

## INVERSES AND SYMMETRIC MATRICES

**Theorem 1.7.4:** If $A$ is an invertible symmetric matrix, then $A^{-1}$ is also symmetric.

## TRANSPOSES AND SYMMETRIC MATRICES

Consider an $m\times n$ matrix $A$ and its transpose $A^T$ (an $n\times m$ matrix). Then $A A^T$ and $A^TA$ are both symmetric.

## TRANSPOSE, SYMMETRY, AND INVERTIBILITY

**Theorem 1.7.5:** If $A$ is an invertible matrix, then $AA^T$ and $A^TA$ are also invertible.