Section 1.5

Elementary Matrices and a Method for Finding A^{-1}

ELEMENTARY MATRICES

An $n \times n$ matrix is called an **elementary matrix** if it can be obtained from the $n \times n$ identity matrix I_n by performing a singled elementary row operation.

ROW OPERATIONS BY MATRIX MULTIPLICATION

Theorem 1.5.1: If an elementary matrix E results from performing a certain row operation on I_m and if A is an $m \times n$ matrix, then the product EA is the matrix that results when the same row operation is performed on A.

INVERSE OPERATIONS

Since a single row operation produced the elementary matrix E, there is a row operation , called an **inverse operation**, that returns E to I.

Row Operation on I That Produces E	Row Operation on E That Reproduces I
Multiply row i by $c \neq 0$	Multiply row i by $1/c$
Interchange rows i and j	Interchange rows i and j
Add c times row i to row j	Add $-c$ times row i to row j

ELEMENTARY MATRICES AND INVERSES

Theorem 1.5.2: Every elementary matrix is invertible, and the inverse is also and elementary matrix.

EQUIVALENT STATEMENTS

Theorem 1.5.3: If A is an $n \times n$ matrix, then the following statements are equivalent; that is, all are true or all are false.

(a) A is invertible.

(b) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(c) The reduced row-echelon form of A is I_n.

(d) A is expressible as a product of elementary matrices.

ROW EQUIVALENCY

Matrices that can be obtained from one another by a finite sequence of row operations are said to be **row equivalent**.

A METHOD FOR INVERTING MATRICES

To find the inverse of an invertible matrix A, we must find a sequence of elementary row operations that reduces A to the identity matrix and then perform the sequence of row operations on I_n to obtain A^{-1}.