Section 1.3

Matrices and Matrix Operations

MATRICES

A **matrix** is a rectangular array of numbers. The numbers in the array are called **entries**.

The **size** of a matrix is described by the number of rows and columns.

Example: A 2×3 matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ -8 & \frac{1}{2} & -7 \end{bmatrix}$$

SCALARS

A **scalar** is a (real) number.

ENTRIES AND THE GENERAL FORM OF A MATRIX

The entry in row i and column j of matrix A is denoted by either

$$a_{ij} \quad \text{or} \quad (A)_{ij}$$

A general 3×2 matrix is given by

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

SQUARE MATRICES

A is a **square matrix of order n** if it has n rows and n columns. The entries $a_{11}, a_{22}, \ldots, a_{nn}$ are said the be on the **main diagonal** of A.

If A is a square matrix, then the **trace of A**, denoted by tr(A), is the sum of the entries on the main diagonal.

If A is not square, the trace is undefined.

ROW AND COLUMN MATRICES

Row matrices and column matrices (also called **row vectors** and **column vectors**) are of special importance, and it is common practice to denote them by boldface lowercase letters. A general $1 \times n$ row matrix **a** and a general $m \times 1$ column matrix **b** would be written as

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

EQUALITY OF MATRICES

Two matrices are **equal** if they have the same size and corresponding entries are equal.

ADDITION AND SUBTRACTION OF MATRICES

If A and B are matrices of the same size, then the **sum** $A + B$ is the matrix obtained by adding the entries of B to the corresponding entries of A.

The **difference** $A - B$ is the matrix obtained by subtracting the entries of B to the corresponding entries of A.

Matrices of different sizes cannot be added or subtracted.

SCALAR MULTIPLICATION

If A is any matrix and c is any scalar, then the (**scalar**) **product** cA is the matrix obtained by multiplying each entry of A by c. The matrix cA is said to be a **scalar product** of A.

MATRIX MULTIPLICATION

If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the (**matrix**) **product** AB is an $m \times n$ matrix with entries determined as follows. To find entry $(AB)_{ij}$, single out row i from matrix A and column j from matrix B. Multiply the corresponding entries from the row and column together and then add up the resulting products.

MATRIX FORM OF A LINEAR SYSTEM

Given: $x + 2y = 1$
$x - y = 2$

The matrix form of this linear system is

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Symbolically, we would write

$$A\mathbf{x} = \mathbf{b}.$$

The matrix A is called the **coefficient matrix**.

TRANSPOSE OF A MATRIX

If A is an $n \times m$ matrix, then the **transpose of A**, denoted by A^T, is defined to be the $m \times n$ matrix that results by interchanging the rows and columns of A; that is, the first column of A^T is the first row of A, the second column of A^T is the second row of A, and so forth.