

Section 1.1

Introduction to Systems of Linear Equations

LINEAR EQUATION

A linear equation is an equation with variables to the first power only. It can be expressed as

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

EXAMPLES:

1. $2x + 5y = 3$

2. $x_1 + 3x_2 - 2x_3 = 12$

A solution to a linear equation is a set of numbers that makes the equation true. These may involve parameters.

HOMOGENEOUS LINEAR EQUATION

A linear equation is called homogeneous if it can be expressed as

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = 0.$$

EXAMPLE:

$$x - 2y - 3z + w = 0$$

SYSTEMS OF LINEAR EQUATIONS

A system of linear equations is a set of at least two of linear equations. We look for a solution that makes all equations true at the same time.

Example: $x_1 + 2x_2 = -1$
 $x_1 + 2x_2 - x_3 = -4$

Note that $x_1 = -1$, $x_2 = 0$, and $x_3 = 3$ is a solution. The solution could also be expressed as $(-1, 0, 3)$.

SOLUTIONS TO A SYSTEM

- If a system of equations has no solution, then it is called inconsistent.
- If a system of equations has at least one solution, then it is called consistent.

Every system of equations has either no solution, exactly one solution, or infinitely many solutions.

MATRICES

A matrix is a rectangular array (or table) of numbers.

EXAMPLE:

$$\begin{bmatrix} 3 & 4 & 0 & 5 \\ -1 & 3 & -7 & 6 \\ 2 & -4 & 8 & -9 \end{bmatrix}$$

AUGMENTED MATRICES

An [augmented matrix](#) can be used to write a system of equations. The system

$$x_1 + 2x_2 = -1$$

$$x_1 + 2x_2 - x_3 = -4$$

can be written as

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 2 & -1 & -4 \end{bmatrix}$$

ELEMENTARY ROW OPERATIONS

The same operations we perform on a system of linear equations we can also perform on an augmented matrix. These operations are called [elementary row operations](#).

ELEMENTARY ROW OPERATIONS (CONCLUDED)

| System of Equations | Matrix |
|---|---|
| 1. Multiply an equation by a nonzero constant | 1. Multiply a row by a nonzero constant |
| 2. Interchange two equations | 2. Interchange two rows |
| 3. Add a multiple of one equation to another | 3. Add a multiple of one row to another row |

A USE FOR ROW OPERATIONS

Elementary row operations can be used to solve systems of equations.