Section 6.1

Inverse Functions

ONE-TO-ONE FUNCTIONS

Definition: A function f is called **one-to-one** (or 1-1) if it never takes the same value twice; that is,

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2$$

THE HORIZONTAL LINE TEST

A function is one-to-one if and only if no horizontal line intersects the graph more than once.

INVERSE FUNCTION

Definition: Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \text{if, and only if,} \quad f(x) = y$$

for any y in B.

COMMENTS ON INVERSE FUNCTIONS

- $f^{-1}(f(x)) = x$ for every x in A.
- $f(f^{-1}(x)) = x$ for every x in B.
- domain of f^{-1} = range of f
- range of f^{-1} = domain of f
- Do ***NOT*** mistake f^{-1} for an exponent. $f^{-1}(x)$ does ***NOT*** mean $\frac{1}{f(x)}$.

FINDING THE FORMULA FOR AN INVERSE FUNCTION

To find the formula for the inverse function of a one-to-one function:

Step 1: Replace $f(x)$ with y.

Step 2: Interchange the variables x and y; that is, replace all the x's with y's and all the y's with x's.

Step 3: Solve for y.

Step 4: Replace y with $f^{-1}(x)$.

THE GRAPH OF AN INVERSE FUNCTION

The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$.

INVERSE FUNCTIONS AND CONTINUITY

Theorem: If f is a one-to-one continuous function defined on an interval, then its inverse function is also continuous.

INVERSE FUNCTIONS AND DIFFERENTIABILITY

Theorem: If f is a one-to-one differentiable function with inverse function $g = f^{-1}$ and $f'\big(g(a)\big) \neq 0$, then the inverse function is differentiable at a and

$$g'(a) = \frac{1}{f'\big(g(a)\big)}$$

Alternatively, $(f^{-1})'(a) = \dfrac{1}{f'\big(f^{-1}(a)\big)}$