Section 8-3

Testing a Claim About a Mean

ASSUMPTIONS FOR TESTING CLAIMS ABOUT POPULATION MEANS WHEN σ IS NOT KNOWN

1. The sample is a simple random sample.
2. Either or both of these conditions are satisfied:
   - The population is normally distributed.
   - The sample size is larger than 30; that is, $n > 30$.

TEST STATISTIC FOR TESTING A CLAIM ABOUT A MEAN (WITH σ NOT KNOWN)

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

Critical Values and *P*-values:

- Found in Table A-3
- Degrees of freedom (df) $= n - 1$

PROPERTIES OF THE STUDENT *t* DISTRIBUTION

1. The Student t distribution is different for different sample sizes.
2. The Student t distribution has the same general symmetric bell shape as the normal distribution but it reflects the greater variability (with wider distributions) that is expected with small samples.
3. The Student t distribution has a mean of $t = 0$ (just as the standard normal distribution has a mean of $z = 0$).
4. The standard deviation of the Student t distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a $\sigma = 1$).
5. As the sample size n gets larger, the Student t distribution gets closer to the normal distribution.

CRITICAL VALUE AND *P*-VALUE METHODS

We will use both the critical value method and *P*-value method for hypothesis testing about a mean. The process is the same as that described in the last two sections.

CRITICAL VALUES IN STUDENT *t* DISTRIBUTION

The larger Student t critical value shows that with a small sample, the sample evidence must be ***more extreme*** before we consider the difference is significant

P-VALUE METHOD

- Table A-3 includes only selected values of α.
- Specific *P*-values usually cannot be found.
- Use Table to identify limits that contain the *P*-value.
- The TI-83/84 calculator (and some computer programs) will find exact *P*-values.

P-VALUE METHOD USING THE TI-83/84 CALCULATOR

1. Press **STAT**.
2. Arrow over to **TESTS**.
3. Select **2:T-Test…**.
4. Select the **Inpt** type (**Data** or **Stats**)
5. Enter as μ_0 the value of μ from the null hypothesis.
6. Enter the values for **s** (population standard deviation), $\bar{x}$ (sample mean), and **n** (sample size).
7. Select the type of test. Select either $\neq\mu_0$, $< \mu_0$, or $>\mu_0$. This corresponds to the alternative hypothesis.
8. Highlight **Calculate**, and press **ENTER**.

UNDERLYING RATIONALE FOR HYPOTHESIS TESTING

- If, under a given observed assumption, the probability of getting the sample is very small and the observed event occurs *significantly less than* or *significantly great than* what we typically expect with that assumption, we conclude that the assumption is probably not correct.
- When testing a claim, we make an assumption (null hypothesis) that contains equality. We then compare the assumption and the sample results and we form one of the following conclusions:

UNDERLYING RATIONALE FOR HYPOTHESIS TESTING

1. If the sample results can are *not* significantly low or significantly high when the assumption (null hypothesis) is true, we attribute the relatively small discrepancy between the assumption and the sample results to chance.
2. If the sample results *are* significantly low or significantly high when that assumption (null hypothesis) is true, we explain the relatively large discrepancy between the assumption and the sample by concluding that the assumption is not true.

ASSUMPTIONS FOR TESTING CLAIMS ABOUT POPULATION MEANS WHEN σ IS KNOWN

1. The sample is a simple random sample.
2. The population standard deviation σ is known.
3. Either or both of these conditions are satisfied:
   - The population is normally distributed.
   - The sample size is larger than 30; that is, $n > 30$.

TEST STATISTIC FOR TESTING A CLAIM ABOUT A MEAN WHEN σ IS KNOWN

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

Critical Values and *P*-values: Found using the standard normal (z) distribution in Table A-2.

P-VALUE METHOD USING THE TI-83/84 CALCULATOR

1. Press **STAT**.
2. Arrow over to **TESTS**.
3. Select **1:Z-Test...**.
4. Select the **Inpt** type (**Data** or **Stats**)
5. Enter as μ_0 the value of μ from the null hypothesis.
6. Enter the values for **σ** (population standard deviation), $\bar{x}$ (sample mean), and **n** (sample size).
7. Select the type of test. Select either $\neq\mu_0$, $< \mu_0$, or $>\mu_0$. This corresponds to the alternative hypothesis.
8. Highlight **Calculate**, and press **ENTER**.